

Quantity Theory of Money Problem Set

Brad DeLong
Marc Gersen

We have our quantity theory of money equation, with velocity explicitly depending on the inflation rate π :

$$MV_0 e^{v_i \pi} = PY$$

We're going to use lower-case letters to indicate logs, so we will also work with:

$$m + v = p + y$$

$$v = v_0 + v_i \pi$$

We have this rate-of-change version of the quantity theory as well:

$$\frac{dm}{dt} + \frac{dv}{dt} = \pi + \frac{dy}{dt}$$

And let's call output-adjusted and trend-velocity-adjusted money $m-y+v_0$ by the label μ :

$$\pi = \frac{dp}{dt} = \frac{p - \mu}{v_i}$$

(1) If μ is constant at μ^* , then we can integrate to get:

$$p = \mu^* + Ae^{t/v_i}$$

The only non-explosive solution to which is:

$$p = \mu^*, \text{ or } P = \frac{MV_0}{Y^*}$$

(2) If μ is growing at a constant rate ϕ , then define:

$$x = p - \mu - \phi v_i$$

and find:

$$p = \mu + \phi v_i + Ae^{t/v_i}$$

The only non-explosive solution to which is:

$$p_t = \mu_0 + \phi t + \phi v_t, \text{ or } P_t = \frac{M_0 V_0}{Y_0^*} e^{\phi v_t} e^{\phi t}$$

We have the relationship between ϕ and the rates of money and potential GDP growth given by:

$$\phi = \frac{1}{M} \frac{dM}{dt} - n - g$$

That is all we are going to need.

Consider the following scenario:

(1) After some time t_2 , the nominal money stock M grows and is expected to grow at a proportional rate:

$$\frac{1}{M} \frac{dM}{dt} = \phi_3 + n + g$$

(2) At the moment t_3 , the nominal money stock M suddenly and *unexpectedly* increases its proportional rate of growth from ϕ_1+n+g to ϕ_3+n+g . Between t_3 and some early moment t_2 , however, the proportional rate of growth of the nominal money stock M is and is expected to be constant at ϕ_1+n+g .

(3) At the moment t_2 , the *level* of the nominal money stock M suddenly and *unexpectedly* increases by a proportional factor $k_2 > 1$; the proportional rate of growth, however, does not and is not expected to change.

(4) At some moment t_1 before t_2 , the proportional rate of growth of the nominal money stock M increases and was always expected to increase from $n+g$ to ϕ_1+n+g .

(5) At some moment t_0 before t_1 , the level of the nominal money stock increases and was always expected to increase by a proportional factor k_0 .

Now characterize the behavior of the price level and the inflation rate in the scenario above, in which the economy is buffeted by all of these changes in the pattern of money growth that divides time up into these five periods. Specifically, answer the following questions:

- What is the inflation rate after t_3 ?
- What is the level of $\mu = m - y + v_0$ after t_3 ?
- By what proportional factor does the price level change at time t_3 ?
- What is the inflation rate between t_2 and t_3 ?

- What is the level of $\mu = m - y + v_0$ between t_2 and t_3 ?
- By what proportional factor does the price level change at time t_2 ?
- By what proportional factor does the price level change at time t_1 ?
- If we call the price level at t_1 P_1 , what equation describes the behavior of the price level and the inflation rate before t_1 and after t_0 ?
- By what proportional factor does the price level change at time t_0 ?
- If we call the price level at t_0 P_0 , what equation describes the behavior of the price level and the inflation rate before t_0 ?