The Price Level in the Medium Run:

The Quantity Theory of Money in the Flexible-Price Full-Employment Business-Cycle Model

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The U.S. experienced an episode of relatively mild inflation—prices rising at between five and ten percent per year—in the 1970s. Although relatively mild, that inflation was large enough to cause significant economic and political trauma. Avoiding a repeat of the inflation of the 1970s remains a major goal of economic policy even today, a quarter century later.

Many countries have experienced inflations that are *not* mild. In Russia in 1998 the price level rose at a rate of 60 percent per year. In Germany in 1923 prices rose at a rate of 60 percent per *week.* So-called *hyperinflations* have been seen in many other countries in this century, from Argentina to Ukraine, from Hungary to China. They are extremely destructive. They inflict severe damage on the ability of money to grease the wheels of the social mechanism of exchange that is the market economy. The system of prices and market exchange breaks down, and production can fall to a small fraction of potential output.

In the medium run model we analyze real economic quantities withought ever having to refer to the price level or the inflation rate. This is a special feature of the model, this *classical dichotomy*: the fact that real variables (like real GDP, real investment spending, or the real exchange rate) can be analyzed and calculated without thinking of nominal variableslike the price level. You will also hear economists speak of this as the property that "money" is *neutral,* or that "money" is a *veil*—a covering that does not affect the shape of the face underneath.

Now it is time to turn to the question of the price level and inflation in the medium-run model. This is worth doing for two reasons. First, it provides a useful baseline analysis against which to contrast the conclusions of future chapters. Second, whenever we look over relatively long spans of time—three to five years or more, perhaps—wages and prices *are* effectively flexible, they *do* have time to move in response to shocks, and the flexible-price assumption is a fruitful and useful one.

First, some underbrush: when normal people use the word "money," they may mean a number of things. "Money" may be used as a synonym for wealth: when we say "she has a lot of money," we mean that she is wealthy. "Money" may be used as a synonym for income: when we "he makes a lot of money," we mean that he has a high income. Economists, however, are not normal. When an economist uses the word "money," he or she means something different. To an economist, "money" is wealth that is held in a readily-spendable form. Money is that kind of wealth that you can use immediately to buy things because others will accept it as payment. Today the economy's stock of money is made up of:

(1) Coin and currency, that are transferred by handing the cash over to the seller (which almost everyone will accept as payment for goods and services.

(2) Checking account balances, that are transferred by writing a check (which most people will accept as payment for goods and services).

(3) Other assets—like savings account balances, unspent credit lines, et cetera—that can be turned into cash or demand deposits nearly instantaneously, risklessly, and costlessly.

Why do economists adopt this special definition of money? I do not know. Giving normal household words special definitions is probably a bad thing to do. It causes confusion and misunderstanding. Yet economists do so for not only "money" but also for terms like "investment" and "utility."

Whether assets that can be quickly and cheaply turned into cash like savings account balances, money market mutual funds, liquid Treasury securities, and so on are included in the money stock is a matter of taste and judgment. At what level of cost and inconvenience is an asset no longer "readily spendable"? There is no clear, hard, bright-line, unambiguous answer. Thus economists have a number of different measures of the money stock—identified by symbols like H, M1, M2, M3, and L. We won't go there: this is not the money and banking course.

Money—and its smooth functioning—is essential. Try to imagine a barter economy, an economy without the social convention of money. In our world all you need to carry out a market transaction—whether you want to buy or sell some good or service—is to either have money (if you want to buy) or for the purchaser to have money (if you want to sell). In a barter economy market exchange would require the so-called *coincidence of wants.* You would have to have physically in your possession some good or service that they wanted, and they would have to have in their possession some good or service that you wanted. An extraordinary amount of time and energy would be spent simply arranging the goods one needed to trade.

In the United States the Federal Reserve, the nation's central bank, determines the money stock. That is the basic task of *monetary policy*: the determination of the money stock. The central bank directly determines the *monetary base*, the sum of currency in circulation and of deposits at the Federal Reserve's twelve branches. When the central bank wants to reduce the monetary base it sells short-term government bonds and accepts currency or deposits at its regional branches as payment. The currency is removed from circulation and stored in a basement somewhere; the deposits it receives as payment are then erased from its books. Thus the monetary base declines. When the Federal Reserve wants to increase the monetary base it buys short-term government bonds, paying for them with currency or by crediting the seller with a deposit at the Federal Reserve. These transactions are called *open-market operations,* because the Federal Reserve buys or sells bonds on the open market

The Federal Reserve directly controls the monetary base. The other measures of the money stock are determined by the interaction of the monetary base with the banking sector. Banks accept checking and savings account deposits. They loan out the purchasing power deposited in the bank, earn interest, and provide the depositor with a claim to wealth in readily-spendable form. But central banks limit commercial banks' ability to accept their deposits to a given multiple of their reserves. And financial institutions have an extremely powerful incentive never to get caught without the cash to satisfy depositors' demands. Once again, we don't talk about these issues: this is not the Money and Banking course.

Households have a *demand* for money just as they have a demand for any other good. They want to hold a certain amount of wealth in the form of readily-spendable purchasing power because the stuff is useful. The more money in your portfolio, the easier it is to buy things. Too little money makes living one's life pointlessly difficult. You have to waste time running to the bank for extra cash or waste energy and time liquidating pieces of your portfolio before you can carry out your normal daily transactions.

On the other hand, you don't want to have too much of your wealth in the form of readily-spendable purchasing power. Cash sitting in your pocket is not earning interest at the bank. Wealth you will not want to spend for five years could earn a higher return as a certificate of deposit or invested in the stock market than sitting in your checking account.

We model this through the Quantity Theory of Money Equation:

$$MV = PY$$

M is the money stock. P is the price level. PY is the total nominal flow of spending. For each dollar of total spending on goods and services, households want to hold 1/V dollars in “money.” That’s their sweet spot—that’s when the interest they are giving up by holding more “money” is just offset by the additional convenience they get. We call “V” complicated!)--is the *velocity* of money. The velocity of money is a measure of how "fast" money moves through the: how many times a year the average unit of money shows up in someone's income and is then used in to buy a final good or service that counts in GDP.

From one point of view, the quantity theory equation is an identity: we have no way to measure velocity, and so we define “velocity” as whatever is needed to make the quantity theory true. The quantity theory becomes a theory rather than an identity only when we come up with a *model* of velocity, with a behavioral relationship that tells us what the velocity of money is.

Economic theory suggests that money demand should be *inversely* related to the nominal interest rate—sum of the real interest rate and the current inflation rate. The cash in your purse or wallet does not earn interest. Your checking account balances earn little or no interest as well. As a result, their purchasing power over real goods and services erodes at the rate of inflation. The expected real return on keeping your money in readily spendable form is $-\pi$, the negative of the expected inflation rate.

By contrast, were you to take a dollar out of your checking account and invest it, its real return would be the real interest rate r. The difference between the rate of return on money balances and the rate of return on other assets is the *opportunity cost* of holding money. This opportunity cost is the sum of the inflation rate π and the real interest rate r: that is, the nominal interest rate i. The higher is this opportunity cost of holding money, the lower is the demand for money balances.

Thus we model the velocity of money as:

$$V = V_b e^{v_i i}$$

The ratio between spending and money holdings depends on a term V_b that captures the procedures and organization of transactions—how the banking system works, et cetera—that may well change over time, and a term that depends on the nominal interest rate: the higher the nominal interest rate, the larger is velocity. We will usually want to use the definition of the nominal interest rate:

$$i = r + \pi$$

to rewrite the velocity equation as:

$$V_b e^{v_i i} = V = V_b e^{v_i(\pi + r)} = V_b e^{v_i r} e^{v_i \pi} = V_0 e^{v_i \pi}$$

to get an explicit relationship between velocity and inflation.

We believe that causation generally—but not always—runs from the left to the right hand side of the quantity equation. Should the price level be momentarily higher than the quantity equation predicts, households and businesses will notice that they have less wealth in the form of readily-spendable purchasing power than they wish. They will cut back on purchases for a little while to build up their liquidity. As they cut back on purchases, sellers will note that demand is weak and cut their prices, so the price level will fall (or rise less rapidly).

With those preliminaries, we can now analyze what the quantity equation in the form:

$$MV_0e^{v_i\pi} = PY$$

tells us. We're going to use lower-case letters to indicate logs, so we will also work with:

$$m + v = p + y$$
$$v = v_0 + v_i\pi$$

And we have this rate-of-change version as well:

$$\frac{dm}{dt} + \frac{dv}{dt} = \pi + \frac{dy}{dt}$$

To begin, start with the log form:

$$m + v = p + y$$
$$m + v_0 - y + v_i\pi = p$$
$$\pi = \frac{p - (m + v_0 - y)}{v_i}$$

And let's call output-adjusted and trend-velocity-adjusted money m-y+v_0 by the label μ:

$$\pi = \frac{dp}{dt} = \frac{p - \mu}{v_i}$$

(1) If μ is constant at $\mu*$, then we can integrate to get:

$$p = \mu * + Ae^{t/v_i}$$

The only non-explosive solution to which is:

$$p = \mu *, \ or \ \ P = \frac{MV_0}{Y*}$$

(2) If μ is growing at a constant rate ø, then define:

$$x = p - \mu - \phi v_i$$

and find:

$$p = \mu + \phi v_i + Ae^{t/v_i}$$

The only non-explosive solution to which is:

$$p_t = \mu_0 + \phi t + \phi v_i, \ or \ \ P_t = \frac{M_0 V_0}{Y_0 *} e^{\phi v_i} e^{\phi t}$$

with the relationship between ø and the rates of money and potential GDP growth given by:

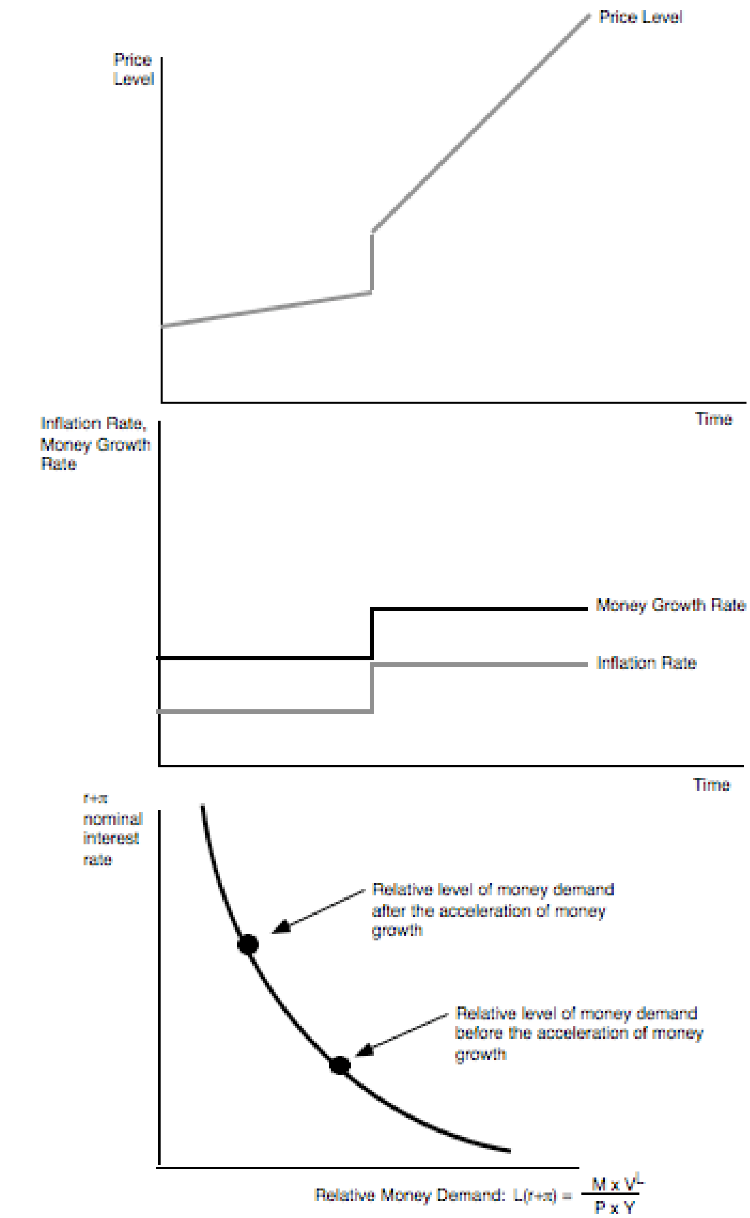
$$\phi = \frac{1}{M}\frac{dM}{dt} - n - g$$

That is all we are going to need to analyze the behavior of the price level.

Let's apply this model heuristically. Consider a situation in which the rate of growth of the money stock is +6% per year and real GDP growth is +3% per year. Then inflation will be 3% per year as well (as long as trend velocity is not changing).

Now suppose that the rate of growth of the money stock suddenly and unexpectedly increases permanently from 6% per year to 10% per year.

Effects of an Increase in Money Growth



When the economy settles down, the new inflation rate will be 4% per year higher—7% instead of 3% per year. But at an inflation rate of 7% per year, the opportunity cost of holding money was higher. If the real interest rate is stable at 3% per year, then the opportunity cost of holding money has just jumped from 6% to 10% per year. A higher opportunity cost of holding money will raise the velocity of money. If the money stock and real GDP remain fixed, this increase in the velocity of money will cause the price level to jump suddenly and discontinuously. The more sensitive is money demand to the nominal interest rate, the larger will be the sudden jump in the price level.

Thus in the flexible-price macroeconomy, an unanticipated change in the rate of growth of the money stock not only changes the long-run inflation rate, it also causes an immediate jump in the price level at the moment that households and businesses become aware that the rate of money growth has changed.

In general, we are going to want to be able to analyze the effects of (a) changes in the level of the money stock and (b) changes in the rate of growth of the money stock that can be (c) anticipated or (d) unanticipated and surprising. But I'm going to leave that to you in the problem set.

For the problem set, consider the following scenario:

(1) After some time t_3, the nominal money stock M grows and is expected to grow at a proportional rate:

$$\frac{1}{M}\frac{dM}{dt} = \phi_3 + n + g$$

(2) At the moment t_3, the nominal money stock M suddenly and *unexpectedly* increases its proportional rate of growth from $ø_1$+n+g to $ø_3$+n+g. Between t_3 and some early moment t_2, however, the proportional

rate of growth of the nominal money stock M is and is expected to be constant at $ø_1$+n+g.

(3) At the moment t_2, the *level* of the nominal money stock M suddenly and *unexpectedly* increases by a proportional factor k_2>1; the proportional rate of growth, however, does not and is not expected to change.

(4) At some moment t_1 before t_2, the proportional rate of growth of the nominal money stock M increases and was always expected to increase from n+g to $ø_1$+n+g.

(5) At some moment t_0 before t_1, the level of the nominal money stock increases and was always expected to increase by a proportional factor k_0.

Now characterize the behavior of the price level and the inflation rate in each of these five periods, answering the following questions:

- What is the inflation rate after t_3?
- What is the level of μ=m-y+v_0 after t_3?
- By what proportional factor does the price level change at time t_3?
- What is the inflation rate between t_2 and t_3?
- What is the level of μ=m-y+v_0 between t_2 and t_3?
- By what proportional factor does the price level change at time t_2?
- By what proportional factor does the price level change at time t_1?
- If we call the price level at t_1 P_1, what equation describes the behavior of the price level and the inflation rate before t_1 and after t_0?
- By what proportional factor does the price level change at time t_0?
- If we call the price level at t_0 P_0, what equation describes the behavior of the price level and the inflation rate before t_0?