

The Value of Investment Opportunities

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Suppose we start with an investor with wealth W and with the opportunity to invest in an asset with return R :

$$R = \begin{cases} 1 + g + \sigma, & p = 0.5 \\ 1 + g - \sigma, & p = 0.5 \end{cases}$$

What is the certainty-equivalent wealth W^* of this investor?

Let us begin with the log utility case. Then the investor invests a share of his or her wealth in the investment:

$$\theta = \frac{g}{\sigma^2 - g^2}$$

And has a certainty-equivalent wealth of:

$$W^* = W \left(\frac{g}{\sqrt{\sigma^2 - g^2}} \right)$$

An alternative way of posing the problem:

Suppose we start with an investor with wealth W and with the opportunity to invest in an asset with return R :

$$R = \begin{cases} 1 + x, & p = 0.5 \\ 1 - y, & p = 0.5 \end{cases}$$

What is the certainty-equivalent wealth W^* of this investor?

Let us begin with the log utility case. Then the investor invests a share of his or her wealth in the investment:

$$\theta = \frac{1}{2y} - \frac{1}{2x}$$

And has a certainty-equivalent wealth of:

$$W^* = W \sqrt{\frac{1}{2} + \frac{x}{4y} + \frac{y}{4x}}$$

Why does this matter? It matters because it is a way of modeling the value to an organization of avoiding bankruptcy—its value as a “going concern.” Organizations are very cautious about making possibly-ruinous bets. Some of this is simple moral hazard: the managers like their relatively cushy jobs. But some of it is more, and this is a way to start modeling that “more”...