Economics 101b; Fall 2000; Problem Set 2 Answers

1. Consider the production function:

$$\frac{Y}{L} = \left(\frac{K}{L}\right)^{0.5} (E)^{1-0.5}$$

a. Suppose E=1, L=100, and K=64; what is output per worker Y/L?

Output per worker is 0.8

b. Suppose E=3, L=196, and K=49; what is output per worker Y/L?

Output per worker is one-half times the square root of 3.

c. If both capital K and labor L double, what happens to total output Y? (Not output per worker Y/L, but total output.)

Total output doubles: this production function has constant returns to scale.

d. Holding E=1, suppose that capital per worker increases from 2 to 4 and then from 4 to 6. What happens to output per worker?

Output per worker rises from 1.41 to 2 and then from 2 to 2.45; diminishing returns begin to set in.

2. Consider an economy in which the depreciation rate is 3% per year, the rate of population increase is 1% per year, the rate of technological progress is 1% per year, and the private savings rate is 16% of GDP. Suppose that the government increases its budget deficit--which had been at 1% of GDP for a long time--to 3.5% of GDP and keeps it there indefinitely.

a. What is the effect of this shift on the economy's steady-state capital-output ratio?

The savings investment rate falls from 15% of GDP to 12.5% of GDP. The steady-state capital-output ratio falls from 3 to 2.5

b. What is the effect of this shift on the economy's steady state growth path for output per worker?

The economy's steady-state growth path will fall—any given level of the efficiency of labor will generate a lower steady-state growth-path level of output per worker. How much lower depends on the diminishing-returns-to-scale parameter α of the production function.

c. Suppose that your forecast of output per worker 20 years in the future had been $100,000. What is your new forecast of output per worker twenty years hence?

It depends on what the value of α, and thus of the growth multiplier λ, is. The ratio of your old and new forecasts is:

$$\frac{(Y_t / L_t)_{new}}{(Y_t / L_t)_{old}} = \frac{\left(\frac{s_{new}}{n+g+\delta}\right)^{\lambda} \times E_t}{\left(\frac{s_{old}}{n+g+\delta}\right)^{\lambda} \times E_t} = \left(\frac{s_{new}}{s_{old}}\right)^{\lambda}$$

We know that $s_{new}/s_{old} = 5/6$. So:

$$\left(\frac{Y_t}{L_t}\right)_{new} = \$100{,}000 \times \left(\frac{5}{6}\right)^{\lambda}$$

3. Consider an economy with the production function:

$$\frac{Y}{L} = \left(\frac{K}{L}\right)^{0.5} (E)^{1-0.5}$$

in which the depreciation rate on capital is three percent per year, the rate of population growth is one percent per year, and the rate of growth of labor-augmenting technology is one percent per year.

a. Suppose that the savings rate is ten percent of GDP. What is the steady-state capital-output ratio?

Two

What is the value of output per worker on the steady-state growth path written as a function of the level of labor-augmenting technology E?

Y/L = 2 x E

b. Suppose that the savings rate is fifteen percent of GDP. What is the steady-state capital-output ratio?

Three

What is the value of output per worker on the steady-state growth path written as a function of the level of labor-augmenting technology E?

Y/L = 3 x E

c. Suppose that the savings rate is twenty percent of GDP. What is the steady-state capital-output ratio?

Four

What is the value of output per worker on the steady-state growth path written as a function of the level of labor-augmenting technology E?

Y/L = 4 x #

4. What happens to the steady-state capital-output ratio if the rate of technological progress increases?

The steady-state capital-output ratio falls. Why? Because output grows faster because of technology, and the cumulated value of past investment does not keep up.

Would the steady-state growth path of output per worker for the economy shift upward, downward, or remain in the same position?

None of the above. At the moment the rate of technological progress increases, the (new) steady-state value of output per worker is below the old value. But the new steady-state growth path is growing faster than the old one grew because of greater technological progress. As you look toward the future, you find the new steady state growth path crossing the old, and continuing upward faster.

5. Discuss--that is, write two paragraphs evaluating--the following proposition: "An increase in the savings rate will increase the steady-state capital output ratio, and so increase both output per worker and the rate of economic growth both in the first few years after the savings rate has increased and in the very long run as well."

An increase in the savings rate does increase the steady-state capital-output ratio, and does raise the level of output per worker along the steady-state growth path. In the first few years after the increase, it is indeed true that output per worker is above what it would otherwise have been, and that the growth rate of output per worker is above what it would otherwise have been as the economy converges to the new, higher steady-state growth path of output per worker.

In the very long run, after the economy has converged to its new steady-state growth path, output per worker is higher than it would have been. But the growth rate is not: remember that in the long run the growth rate of output per worker along the steady-state growth path does not depend on the savings rate, but only on the rate of technological progress g.

6. Would the steady-state growth path of output per worker for the economy shift upward, downward, or remain the same if capital were to become more durable--if the rate of depreciation on capital were to fall?

It shifts upward: a fall in depreciation raises the steady-state capital-output ratio.

7. Suppose that a sudden disaster--an epidemic, say--reduces a country's population and labor force, but does not affect its capital stock. Suppose further that the economy was on its steady-state growth path before the epidemic. What is the immediate effect of the epidemic on output per worker? On the total economy-wide level of output? What happens subsequently?

Provided the epidemic kills people and leaves the capital stock untouched, in its immediate aftermath the higher capital-labor ratio boosts output per worker. The people who survive are better off. Total output, however, falls: the epidemic has not added anything to any factors of production but has subtracted labor, and with fewer resources the economy can produce less total output.

Subsequently the economy—knocked off of its steady-state growth path by the epidemic—converges back to its steady-state growth path. In the long run the level of output per worker will be unaffected by the epidemic.