Michael Kremer's Theory of the Large-Scale Structure of Human History

The Model:

Four equations: a production function (with Y for output, L for labor (or population), and T for natural resources (normalized to one); a knowledge accumulation function; a definition of output per capita (or per worker); and a Malthusian population growth function:

$$Y_t = A_t L_t^{\alpha} T_t^{1-\alpha} = A_t L_t^{\alpha}$$

$$\frac{dA_t}{dt} = \gamma L_t^{\psi} A_t^{\phi}$$

$$y_t = \frac{Y_t}{L_t}$$

$$\frac{dL_t}{dt} = n(y_t) L_t$$

with:

$$n(0) < 0; \quad n(y^*) \geq n(y) > 0; \quad n(\bar{y}) = 0$$

$$n(y) > 0 \quad for \quad y > \bar{y}$$

Purpose of these specifications to guarantee that population falls when y drops too low, and that population rises whenever y is above the steady-state Malthusian subsistence level y-bar.

Solving the Model:

$$\frac{1}{y_t}\frac{dy_t}{dt} = \frac{1}{A_t}\frac{dA_t}{dt} + (\alpha - 1)\frac{1}{L_t}\frac{dL_t}{dt}$$

$$\frac{1}{y_t}\frac{dy_t}{dt} = \gamma L_t^{\psi} A_t^{\phi - 1} + (\alpha - 1) n(y_t)$$

$$\frac{1}{y_t}\frac{dy_t}{dt} = \gamma L_t^{\psi} \left[y_t L_t^{1-\alpha} \right]^{\phi - 1} + (\alpha - 1) n(y_t)$$

$$\frac{1}{y_t}\frac{dy_t}{dt} = \gamma y_t^{\phi - 1} L_t^{\psi - (1-\phi)(1-\alpha)} + (\alpha - 1) n(y_t)$$

This last equation plus:

$$\frac{1}{L_t}\frac{dL_t}{dt} = n(y_t)$$

make up a system we can plot on a phase diagram, with y on the vertical and L on the horizontal axis...