4. Precautionary Savings. Suppose that we have a representative consumer with a time-separable constant relative risk aversion utility, an opportunity to invest funds at a constant net rate of return r, and a constant rate of time discount ρ.

Then consumption satisfies the Euler equation (first order condition):

$$C_t^{-\theta} = \frac{1}{1+\rho} E_t\left[(1+r)C_{t+1}^{-\theta}\right]$$

a. Suppose that r=ρ. Will consumption then follow a random walk with zero drift? Why or why not?

Not really. We have:

$$1 = E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\theta}\right\}$$

which means that the expected value of period t+1 consumption will be equal to the expected value of period-t consumption only if there is no variance in period t+1 consumption. The expected value of period t+1 consumption will be equal to period t consumption only if period t+1 consumption has zero variance. This is not a "random walk" because there is nothing random about it.

b. Suppose that r=ρ. Will consumption then follow a random walk with constant drift? Why or why not?

Not really. Again we have:

$$1 = E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\theta}\right\}$$

Using g to stand for the growth rate of consumption between periods t and t+1, and expanding, we find:

$$1 = E_t\left\{(1+g_{t+1})^{-\theta}\right\} = 1 - \theta E_t(g_{t+1}) + \frac{\theta(\theta+1)E_t(g_{t+1}^2)}{2} - \ldots$$

g does not have a constant mean unless all the other terms on the right-hand side are constant. And even if g had an unchanging one-period-ahead distribution, then the *proportional* growth rate of consumption will have a constant mean. But all moments of the g distribution would have to be constant—and even then it would not be consumption but the *log* of consumption that would follow a random walk.

c. Suppose that the consumer in period t receives news that consumption in period t+1 is going to be much more volatile than previously expected because a lot of news about permanent income will arrive in period t+1. On receiving the news that volatility will be high because news about permanent income will shortly be arriving, does he/she raise or lower period-t consumption? Why?

See above. A CRRA agent has a precautionary savings motive. High volatility raises the second-order term in the equation above, thus raises the expected growth rate of consumption, which means that current consumption must be lower than had the news about volatility not arrived.

d. Derive an approximate expression for the expected growth rate of consumption between period t and t+1, keeping terms that are first order in the variance of period t+1 consumption.

From above:

$$E_t(g_{t+1}) = \frac{\theta + 1}{2}\left(\left[E_t(g_{t+1})\right]^2 + Var_t(g_{t+1})\right) + \ldots$$

with higher-order terms omitted: the expected growth rate of consumption is (roughly) proportional to half the degree of relative risk aversion times the variance of consumption growth.