Suggested Solutions to Problem Set 1

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1. **Shapiro-Stiglitz (Romer, 10.3)** The equilibrium level of unemployment is determined by the labor demand equation (L^D)

$$\bar{e}F'(\bar{e}L) = w \qquad \text{(R 10.40)}$$

and the no-shirking condition (NSC)

$$w = \bar{e} + \left(\rho + \frac{\bar{L}}{\bar{L} - NL}b\right)\frac{\bar{e}}{q}. \qquad \text{(R 10.39)}$$

(a) If the discount rate ρ increases and workers become less patient, NSC will shift upward. This increases the equilibrium levels of unemployment and the real wage (see Figure 1). Intuitively, workers discount the future more, so they are less deterred by the prospect of becoming unemployed in the future when caught shirking. To prevent shirking, higher real wages are needed to increase the loss when the worker is fired. This leads to higher unemployment; in addition, it reduces the probability that workers find jobs once unemployed.

(b) A decline in the rate of job destruction b will shift NSC downward, reducing the real wage and decreasing the level of unemployment (see Figure 2). Intuitively, since workers are less likely to lose their job in the absence of shirking, the value of being employed increases. This reduces the real wage and the level of unemployment that is needed to deter shirking.

(c) If the labor force $\bar{L}$ grows, NSC shifts downward. This reduces the real wage and it increases the level of employment but less than the increase in the labor force (see Figure 3). Hence, the level of unemployment increases. Intuitively, with a larger labor force, the unemployed remain jobless for a

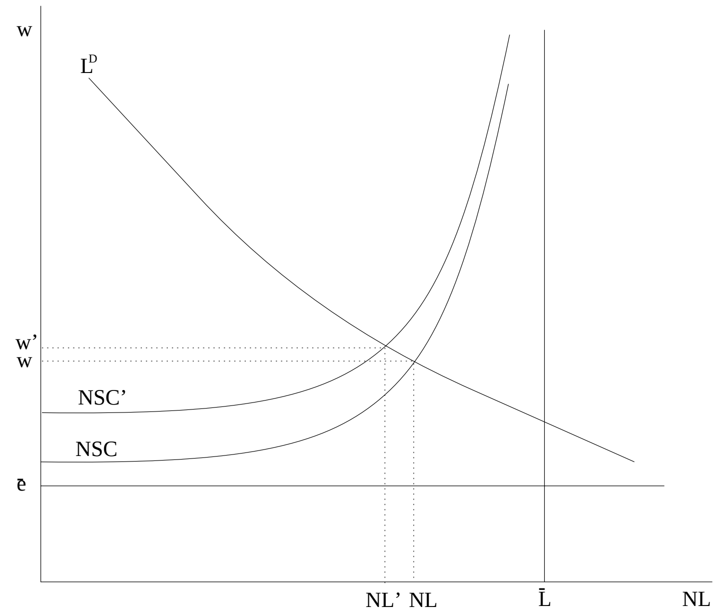

Figure 1: An increase in the discount rate

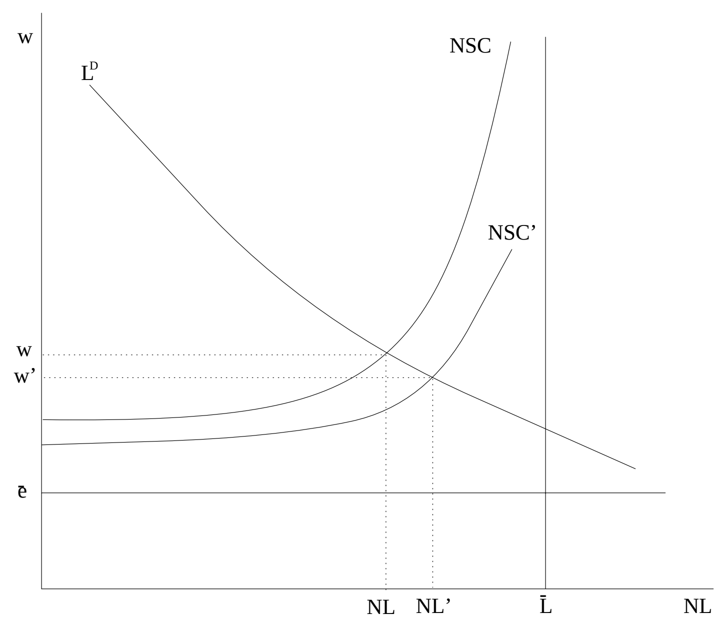

Figure 2: An decline in the job destruction rate

longer time. So, the real wage that is required to prevent shirking is lower. This increase employment, partially offsetting the initial increase in unemployment.

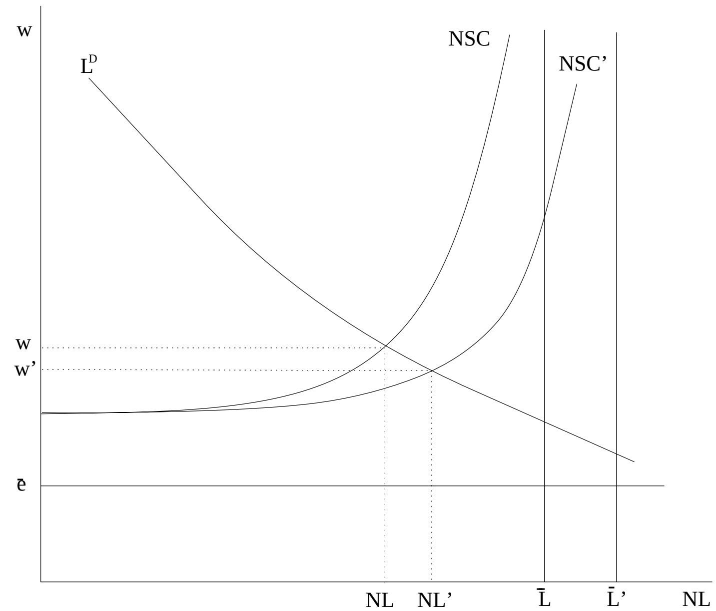

Figure 3: An increase in the labor force

(d) An increase in factor productivity shifts L^D upward. The real wage rises and the level of unemployment drops (see Figure 4).

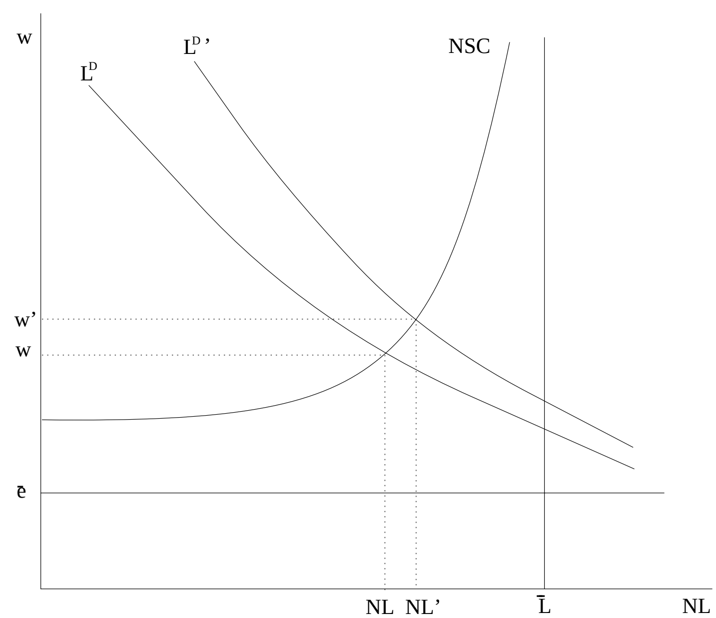

Figure 4: An increase in productivity

Intuitively, workers have become more productive so the firm would like to hire more workers. This leads to a higher wage, which in turn reduces the level of unemployment that is needed to deter shirking.

2. **Fair Wage (George Akerlof and Janet Yellen (1990))**

   (a) Profit maximizing behavior by the representative firm amounts to the Solow condition
   $$\frac{we'(w)}{e(w)} = 1. \qquad \text{(R 10.8)}$$
   Using the effort function $e(w) = \min\left\{\frac{w}{w^*}, 1\right\}$ and taking the fair wage w^* as given, (1) is satisfied for any real wage $0 < w \leq w^*$, so $e(w) = w/w^*$. Note that w is bounded below assuming a finite labor force L^*. In particular, the optimal level of employment for the representative firm is determined by $F'(\frac{w}{w^*}L) = w^*$. Assuming there are N such firms, the lower bound $\underline{w}$ is defined by $F'(\frac{w}{w^*}\frac{L^*}{N}) \equiv w^*$, so that $\underline{w}= Nw^*F'^{-1}(w^*)/L^*$. Hence, the optimal range of real wages is $w \in \left[Nw^*F'^{-1}(w^*)/L^*, w^*\right]$.

   (b) First, suppose firms pay the highest wage in the optimal range so that the average wage equals $\bar{w} = w^*$. Then the specification for the fair wage,
   $$w^* = \bar{w} + a - bu \qquad (1)$$
   implies that $u = a/b$. Presuming $b > 0$, there is positive equilibrium unemployment if $a > 0$. Now, suppose that firms pay a wage of $w^* - d$, where $0 < d \leq w^* - \underline{w}$, so that $\bar{w} = w^* - d$. Then, (1) implies $u = \frac{a-d}{b}$, and unemployment is positive if $a > d > 0$. In words, if the 'fairness markup' over the average wage is larger than the reduction in wages compared to the fair wage, then there will be unemployment.

   (c) The equilibrium generates full employment if $a = d \geq 0$. Note that a full employment equilibrium ($u = 0$) with $a < 0$ is not possible as (1) would imply $w^* < \bar{w}$, which is outside the optimal wage range.

3. **Unemployment in the City (John Harris and Michael Todaro (1970))**
   Let $\bar{N}$ denote the labor force, and N_U and N_R the number of workers employed in the urban and rural sector, respectively. The level of unemployment is denoted by $X \equiv \bar{N} - N_U - N_R$.

(a) The probability of getting an urban job equals $N_U/(\bar{N} - N_R)$. Risk neutrality implies that the expected payoff is equal in both sectors:

$$\frac{N_U}{\bar{N} - N_R} U = R.$$

Rearranging yields $N_R = \bar{N} - N_U \cdot U/R$. Substituting N_R into the definition of unemployment X gives

$$X = \frac{U - R}{R} N_U.$$

In the presence of unemployment $X > 0$, we must have $U > R$.

(b) An increase in the number of jobs in the urban sector, N_U, increases the level of unemployment, X. Formally, $\frac{\partial X}{\partial N_U} = \frac{U-R}{R} > 0$. The increase in urban employment improves the odds of getting a job in the high-paying urban sector. This attracts more rural workers to the city, adding to the pool of unemployed workers. In fact, the rise in unemployment could even exceed the increase in the number of urban jobs if the wage differential is sufficiently large ($U > 2R$).

(c) The probability of being jobless in the city and receiving an unemployment benefit B ($R < B < U$) equals $\left(\bar{N} - N_U - N_R\right) / \left(\bar{N} - N_R\right)$. Risk neutrality implies

$$\frac{N_U}{\bar{N} - N_R} U + \frac{\bar{N} - N_U - N_R}{\bar{N} - N_R} B = R.$$

Rearranging and solving for N_R yields $N_R = \bar{N} - \frac{U-B}{R-B} N_U$. Substituting into the definition of X gives

$$X = \frac{U - R}{R - B} N_U.$$

An increase in unemployment benefits B increases the level of unemployment X. Formally, $\frac{\partial X}{\partial B} = \frac{U-R}{(R-B)^2} N_U > 0$. The more generous unemployment benefits attract more workers to the urban sector, thereby increasing unemployment.

4. **Search** Denote the probability density function and the cumulative density of the uniform wage distribution by $f(w) = \frac{1}{U-L}$ and $F(w) = \frac{w-L}{U-L}$, for $L \leq w \leq U$, respectively.

(a) A worker will only decide to start searching for a job if the search cost is less than the expected wage, i.e. $S \leq E[w] = \frac{U+L}{2}$. Suppose a worker just sampled a job with wage $\hat{w}$. Let V denote the expected value of further searching. Then the worker will decide to accept the offer if $\hat{w} > V$. The expected value of a continued job search equals the expected value of the wage offer the worker will accept minus the expected cost of searching for an acceptable job. So, $V = E[w|w > V] - E[n]S$. Note that $\Pr\{w > V\} = 1 - F(V)$ so that $E[w|w > V] = \int_V^U w \frac{f(w)}{1-F(V)} dw$ and $E[n] = 1 + F(V) + F(V)^2 + \ldots = \frac{1}{1-F(V)}$. Hence,

$$V = \int_V^U w \frac{f(w)}{1 - F(V)} dw - \frac{S}{1 - F(V)}. \tag{2}$$

Alternatively, we could apply dynamic programming and write

$$V = F(V)(V - S) + (1 - F(V)) \left(\int_V^U w \frac{f(w)}{1 - F(V)} dw - S \right). \tag{3}$$

The first term on the right-hand side is the probability of sampling a bad job, $F(V)$, times the difference between the expected value of further searching, V, and the incurred search cost, S. The second term equals the probability of sampling a good job, $1 - F(V)$, times the difference between the expected value of an acceptable wage offer, $E[w|w > V]$, and the incurred search cost, S. Rearranging (3) gives (2), so both approaches are equivalent.

Using (2),

$$V = \int_V^U \frac{w}{U - V} dw - \frac{U - L}{U - V} S = \frac{\frac{1}{2}(U^2 - V^2) - (U - L)S}{U - V}.$$

Rearranging gives

$$\frac{1}{2}V^2 - UV + \frac{1}{2}U^2 - (U - L)S = 0.$$

Solving this quadratic yields

$$V = U - \sqrt{2(U - L)S},$$

where we discarded the solution for which V would exceed the highest possible wage offer U. As a consequence, a worker will accept a newly-sampled job if the wage satisfies

$$\hat{w} \geq U - \sqrt{2(U - L)S}.$$

Notice that in the absence of search costs ($S = 0$), the worker will sample jobs until the highest possible wage U is offered.

(b) No, a worker would never go back to accept a previously sampled job. Since the worker is fully informed about the wage opportunities $f(w)$ and does not learn any new information by sampling jobs, the expected value of further searching (V) is independent of previously sampled jobs. If a worker rejects a job with wage $\hat{w}$, then it must be that $\hat{w}$ is less than V and will always be so. Thus it will never be optimal to accept the job paying $\hat{w}$.

5. **Efficiency of Search Equilibrium (Romer, 10.17)**

(a) Using the matching function (R 10.68) and the steady-state condition (R 10.69) we obtain

$$M(U,V) = KU^{\beta}V^{\gamma} = bE.$$

Rearranging and substituting $U = \bar{L} - E$ and $V = N - E$ gives

$$K\left(\bar{L} - E\right)^{\beta}(N - E)^{\gamma} = bE. \tag{4}$$

This equation defines $E(N)$. Applying implicit differentiation with respect to N and using (4),

$$-\beta b \frac{E}{\bar{L} - E}\frac{\partial E}{\partial N} + \gamma b \frac{E}{N - E}\left(1 - \frac{\partial E}{\partial N}\right) = b\frac{\partial E}{\partial N}.$$

Rearranging gives

$$\begin{aligned} \frac{\partial E}{\partial N} &= \frac{\gamma \frac{E}{N-E}}{1 + \beta\frac{E}{\bar{L}-E} + \gamma\frac{E}{N-E}} \\ &= \frac{\gamma E\left(\bar{L} - E\right)}{(N - E)\left(\bar{L} - E\right) + \beta E(N - E) + \gamma E\left(\bar{L} - E\right)}. \end{aligned} \tag{5}$$

Note that $\partial E/\partial N > 0$. An increase in the total number of jobs means in rise in the number of vacancies for a given number of filled positions. This improves matching between unemployed workers and vacancies, and thereby boosts the number of employed workers.

(b) Social welfare is equal to

$$W(N) = AE(N) - NC,$$

so the marginal effect of an increase in the number of jobs on social welfare is

$$\frac{\partial W}{\partial N} = A\frac{\partial E}{\partial N} - C.$$

Substituting (5) gives

$$\frac{\partial W}{\partial N} = \frac{\gamma E\left(\bar{L} - E\right)}{(N - E)\left(\bar{L} - E\right) + \beta E(N - E) + \gamma E\left(\bar{L} - E\right)}A - C. \qquad (6)$$

Using the simplifying assumption that $r = 0$ and the fact that $V_V = 0$, (R 10.82) implies

$$C = \frac{\alpha A}{a + \alpha + 2b}.$$

Substituting $a = bE/U$ and $\alpha = bE/V$ into C, and $U = \bar{L} - E$, $V = N - E$ and C into (6) produces

$$\begin{aligned}\frac{\partial W}{\partial N} &= \frac{\gamma EU}{VU + \beta EV + \gamma EU}A - \frac{bE/V}{bE/U + bE/V + 2b}A \\ &= \left\{\frac{\gamma EU}{VU + \beta EV + \gamma EU} - \frac{EU}{EV + EU + 2UV}\right\}A.\end{aligned}$$

(c) Regarding the effect on social welfare, notice that $\operatorname{sgn}(\partial W/\partial N)$ equals $\operatorname{sgn}(\gamma EU(EV + EU + 2UV) - EU(VU + \beta EV + \gamma EU))$, which simplifies to $\operatorname{sgn}((\gamma - \beta)E + (2\gamma - 1)U)$, or $\operatorname{sgn}\left((1 - \gamma - \beta)E + (2\gamma - 1)\bar{L}\right)$. So, an increase in the equilibrium number of jobs will increase social welfare if

$$(1 - \gamma - \beta)E > (1 - 2\gamma)\bar{L}.$$

In the special case in which the matching function exhibits constant returns to scale ($\gamma + \beta = 1$), social welfare rises if $\gamma > 1/2$. In the special case of $\gamma = 1/2$, decreasing returns in matching ($\gamma + \beta < 1$) are required. In both cases, vacancies play a relatively greater role in matching than unemployment ($\gamma > \beta$). As a result, the equilibrium number of jobs is inefficiently low and the decentralized equilibrium gives rise to unemployment that is inefficiently high.