Economics 202b: "Equity Premium" Handout

If we start with a constant-relative-risk-aversion utility function, our first-order condition is:

$$1+\rho = E_t\left[(1+r^i_{t+1})(1+g^c_{t+1})^{-\theta}\right]$$

where r is the rate of time preference, r^i_{t+1} is the return on asset i, g is the rate of growth of consumption, and θ is the coefficient of relative risk aversion. If we take a Taylor expansion and throw away second order terms, we get:

$$E_t(r^i_{t+1}) = \rho + \theta E_t(g^c_{t+1}) + \theta Cov(r^i_{t+1}, g^c_{t+1}) - \frac{\theta(\theta+1)}{2} Var(g^c_{t+1})$$

For a risk-free asset paying r*, this becomes:

$$r*_{t+1} = \rho + \theta E_t(g^c_{t+1}) - \frac{\theta(\theta+1)}{2} Var(g^c_{t+1})$$

And so the difference between the expected rate of return on a risky asset and the risk-free rate is:

$$E_t(r^i_{t+1}) - r*_{t+1} = \theta Cov(r^i_{t+1}, g^c_{t+1})$$

Mehra and Prescott (1985) apply this relationship to the U.S. stock and bond markets over this century. For the United States over the past century the *equity premium* (diversified common stocks vs. short-term Treasury bonds) is 6 percentage points per year. The standard deviation of the growth of (nondurable) consumption is 3% per year; the standard deviation of the return on a diversified portfolio of equities is 15% per year. And the correlation between these is about .5—implying a θ of 27 in the utility function.

Such a utility function is steeply, steeply curved: such an agent would prefer to give up 48.6% of his income than to enter a lottery which would have a 50% chance of boosting his or her consumption by 50%, and a 50% chance of reducing his or her consumption by 50%.

Such a steeply, steeply curved utility function is hard to reconcile with a low risk-free rate of about 1% per year. Substituting the high coefficient of relative risk aversion back into the consumption growth equation produces:

$$r*_{t+1} = \rho + 1.105$$

That's not 1.105%--that's 1.105. There are no parameters of the utility function that can produce a risk-free rate of 1% per year.

To put it another way, with an equity premium of 6% per year and a standard deviation of equity returns of 15% per year, then:

…after one year, the odds that stocks have outperformed Treasuries is:	66%
…after two years, the odds that stocks have outperformed Treasuries is:	71%
…after five years, the odds that stocks have outperformed Treasuries is:	81%
…after ten years, the odds that stocks have outperformed Treasuries is:	88%
…after twenty years, the odds that stocks have outperformed Treasuries is:	96%
…after thirty years, the odds that stocks have outperformed Treasuries is:	99%

Only if you give an insanely great weight to those states of the world in which the stock market collapses and stays low does it make sense for any moderately long-term investor to put money in short-term Treasuries rather than diversified equities.