Implicit Contracts and Underemployment Equilibria

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This paper studies an industry with demand uncertainty which prompts risk-neutral firms to act both as employers and as insurers of homogeneous, risk-averse laborers. The resulting contractual arrangements turn out, in their simplest form, to be more likely to specify full employment the more of the following conditions prevail: small variability in product price, above-average economy-wide labor demand, highly risk-averse workers, small unemployment compensation, and highly competitive product market. Otherwise, it may be optimal for firms to lay off, by random choice, part of the work force during low states of demand.

I. Introduction

Competitive wage theory predicts that firms will adjust to a contraction in product demand by lowering both employment and the real wage rate, in contradiction to the normal industrial practice of laying off unneeded workers and paying unchanged wages to the rest of the work force. This paradox,[1] which arises from the close relationship between competitive equilibria and Pareto optima in auction markets, spawned a

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[1] To quote from Arrow (1972): "One of the mysterious things is why [the employers] do not cut wages. It may be that they really do not know what is going on in terms of the possible need to rehire labor in the future and are worried about the impact of any wage cuts on their ability to proceed in the labor market later."

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substantial corrective effort by economic theorists. One reaction, exemplified in the writings of Keynes (1936) and of a diverse group of epigones (Patinkin 1956; Clower 1966; Leijonhufvud 1968; Barro and Grossman 1971), has been to abandon short-run market clearing as a workable assumption and focus instead on quantity adjustments as the primary means of response to market disturbances.

A far less systematic, but intuitively more transparent, commentary[2] has appeared periodically in the literature, arguing in effect that deviations from Pareto optimality occur because continuous-auction models cannot do justice to certain attributes peculiar to the labor resource. Chief among these attributes are the limited mobility of the labor resource and the difficulty of diversifying human capital under uncertainty.

The relative immobility of labor over markets, especially those involving different industries or geographical locations, is fairly well documented.[3] However, the obvious fact bears repeating that no markets exist for the direct exchange of claims on future labor services;[4] the costs of monitoring and enforcement, and "moral hazard," are some reasons why such markets have not arisen.[5]

Nevertheless, at least part of the risk an uncertain labor income stream creates for its recipient can be shifted to third parties by employee intermediation, that is, by the tacit or open commitment of the firm to guarantee its personnel that their wage rates, hours worked, employment status, or a combination of all such factors, will be in some degree independent of the vicissitudes of the business cycle.[6] The risk is thereby transferred from wages to profits and, via the capital market, to the income streams of the firm's owners and creditors.

This process is subject to two limitations. First, assurances of the sort just described will not be handed out evenly to all personnel. In breadth and firmness of commitment, employers will discriminate in favor of persons in whose training substantial investment has been or is about to be made. Others who are thought to possess superior qualities of productivity, reliability, adaptability, etc., will be similarly favored. Second, a rational entrepreneur will not shield the terms of employment of even his most valuable group of employees against arbitrary demand fluctuations if doing so involves more than a token probability that the firm might go bankrupt.

Neither qualification should be a hindrance in the short-run, homo-

[2] For interesting early specimens, see Knight (1921, chap. 9) and Hicks (1932, chap. 3).

[3] Cf. the evidence in Parker and Burton (1971).

[4] Unions could conceivably serve that function were it not generally the case that incomes of the members of any single union are highly correlated.

[5] This discussion draws heavily on Brainard and Dolbear (1971); see also Tobin (1972) and Gordon (1974).

[6] It is in this sense that Knight (1921, pp. 271–72) characterizes wages as "contractual" income and profits as "residual" income.

geneous-labor problem that is studied in this paper; it is sufficient that, in the course of the relationship between employer and employees, enough scope exists for the former to unburden the latter of at least some of the variability that otherwise would accompany wage income. At first approximation, one need not worry whether risk shifting occurs merely because the firm's net worth far exceeds that of its average employee or because it is in the nature of hiring that "the confident and venturesome 'assume the risk' or 'insure' the doubtful and timid by guaranteeing to the latter a specified income in return for an assignment of the actual results" (Knight 1921, pp. 269–70).

The drift of the preceding arguments points to a more complex view of the labor market than is customary in conventional short-run analyses: in uncertainty, labor services are not auctioned off in quite the same way fresh fruit is. Rather, they are exchanged for some implicit set of commitments, hereinafter called an *implicit labor contract*, on the part of the firm to employ the owner of those labor services for a "reasonable" period of time and on terms mutually agreed upon in advance.

In such a market, job choice will depend on what subjective value prospective employees place on alternative contracts, and not on the value of *any one* contract component. After all available contracts have been compared, and before the state of demand in any industry is known, suppliers of labor services will each gravitate toward the highest attainable value. The initial distribution of the labor force thus achieved will be independent of the state of nature; it will also tend to have some permanence, as indicated by the large proportion of rehires in manufacturing accessions.

If contracts, then, are not easily abrogated, employers should possess considerable discretion over specific terms of employment. The possibility arises, for instance, that a job which requires the same technical skills as another will pay a different wage. The freedom to set specific components of contracts at a level above or below what prevails elsewhere is limited by the familiar property of any equilibrium distribution of the labor force over firms: no economic agent values the bargain he/she has actually agreed to less than any other feasible contract.[7]

A systematic study of these issues begins in Section II with a stochastic environment in which risk-neutral entrepreneurs and identical, risk-averse workers with indivisible leisure endowment exchange a particularly simple type of contract. Section III examines the conditions under which the wage schedules of labor contracts will be invariant to changes in relative product price. A necessary and sufficient condition for the sub-optimality of full-employment contracts is developed and interpreted in Section IV. A complete characterization of the optimal contract follows

[7] Cf. Stiglitz (1974).

in Section V along with a comparison of its provisions with what would prevail in a labor acution market beset by identical stochastic disturbances. A summary and conclusions appear in Section VI.

II. Uniform Contracts

Suppose that the quantity of a perfectly homogeneous, nonstorable commodity demanded of an industry depends on the prevailing price, p, of the commodity relative to that of all other goods (the fixed, exogenous "price level"), on the state of nature, s, and, possibly, on a vector of nonrandom, nonprice variables over which the industry has no control. For simplicity, assume that the actual state is drawn randomly from a set of discrete states $S = \{s | s = s_1, s_2, \ldots, s_J\}$ according to a known probability distribution $q(s)$, for which

$$\sum_{s \in S} q(s) = 1. \tag{1}$$

The industry consists of a fixed number of identical, perfectly competitive, risk-neutral firms to which changes in s are revealed through shifts in the product price they can charge their customers. Let $p(s)$ be the (parametrically given) mapping of states into prices and, without loss of generality, suppose that, for any s_1, s_2 in S,

$$s_1 < s_2 \Leftrightarrow p(s_1) < p(s_2). \tag{2}$$

On the other side of the labor market there is an even larger number, M, of risk-averse workers who are identical[8] in tastes, endowments, and technical ability and who may or may not have alternatives outside this industry. Assume, for simplicity, that these workers command one indivisible unit of leisure and that their preferences over consumption, c, and leisure, l, are embodied in a monotone, bounded utility function $v(c, l)$, concave in c.[9]

Let the random variable y denote current labor income; the constants α and c_0 denote, respectively, current property income and the maximum level of consumption attainable when $y = 0$ and $l = 1$. That level may exceed α by the amount, if any, of unemployment compensation which accrues to the worker. Without loss of generality, we set

$$v(c_0, 0) = 0 \text{ and } v(c_0, 1) = K \tag{3}$$

[8] This assumption serves to keep down the dimensionality of the contract space. For instance, if workers differed systematically in preferences or in their endowment of human or nonhuman wealth, then contracts indexed on all of these characteristics would generally be appropriate.

[9] Observed labor preferences for lumpy leisure streams (long weekends, paid vacations, etc.) raise a strong suspicion that v is not concave in l for high enough values of leisure consumption. The implications for unemployment of preferences locally nonconvex in leisure are examined in Sec. IV.

and define an increasing, concave function $u(\cdot)$ from

$$u(y) \equiv v(\alpha + y, 0). \tag{4}$$

The range of contracts offered by each firm will depend partly on how varied its work force is in skill, attitude toward risk, and the like, and partly on the length and the administrative costs of striking, monitoring, and enforcing each agreement. The most important nonwage component of such agreements is probably "job security," which depends crucially on variations in employment.

Suppose, then, for simplicity, that all freely agreed upon contracts are strictly enforceable, perhaps because "cheaters" suffer a catastrophic loss in reputation; are of the same, institutionally fixed duration; and pay no direct compensation to unemployed workers. Since workers are, by assumption, absolutely identical, a *uniform* contract will likely be offered to all of them. This will be a random vector of the form

$$\delta = \{w(s), n(s)\}, \tag{5}$$

which reveals what wage (relative to the price level) and volume of employment the typical firm plans to offer in each state.

Valuation and Dominance

The valuation of a contractual offer by laborers is clearly contingent on the ultimate size, m, of the firm's work force. The larger the number of people who consent to a given contractual offer before the state of nature is known, the smaller the (equal for all) probability, $\rho(s)$, that any of them will end up employed in state s. In fact,

$$\rho(s) = \min [1, n(s)/m]. \tag{5a}$$

Let $V(\delta, m)$ be the expected utility from a contract δ which attracts m workers, where the expectation in V should be taken with respect to both s *and* employment status. Recall that the utility of working in s is $u[w(s)]$ and of not working is K. Hence,

$$V(\delta, m) = E_s[\rho u(w) + (1 - \rho)K]. \tag{5b}$$

From (5a) and (5b) it follows directly, as one might expect of a competitive labor market with mobile laborers of known quality, that all vacancies will be filled.[10] Hence,

$$\rho(s) = n(s)/m \qquad \forall s. \tag{5c}$$

[10] This is a statement about averages. It does not mean to deny that the firm could profitably use more than the contracted amount of labor services in a particularly favorable state of nature but, rather, that it cannot do so on the basis of information available at the time contracts are struck.

The actual value of m corresponding to any contract offer will depend, naturally, on market alternatives. Let $\lambda(K < \lambda)$ be the market valuation attached to such offers under an equilibrium distribution of the labor force. Then the right-hand side of (5b) must equal λ, and m is a function of both λ and the components w and n of the contract.

Some useful nomenclature follows. In *definition* 1, a contract $\delta = \{w(s), n(s)\}$ is called (i) *feasible* if $V(\delta, m) = \lambda$ for some positive m such that $n(s) \leq m$ for all s; (ii) *full employment* (underemployment) if $n(s) = m$ for all s ($< m$ for some s); and (iii) *fixed wage* (variable wage) if w is (is not) independent of s. An example of an infeasible contract is $n(s) > 0$ for some s, $Eu(w) < K$. A Paretian criterion for comparing contracts follows. *Definition* 2: Let m be the size of the work force attracted and $\pi(\delta_1)$ be the expected profit accruing to a firm under a feasible contract, δ_1. Then δ_1 is said to (i) *dominate* $\delta_2(\delta_1 \gtrsim \delta_2)$ if δ_2 is feasible for m and $\pi(\delta_1) \geq \pi(\delta_2)$; and (ii) be *optimal* if it dominates all feasible contracts.

These definitions suggest that the optimum contract, δ^*, is simply the solution to a well-defined constrained maximum problem. The primary aim of this paper, however, is not to characterize δ^* fully (which is done in Sec. V) but, rather, to find out whether δ^* is a full-employment contract. In Section III I reduce the dimensionality of the firm's opportunity set by excluding contracts that are demonstrably inferior.

III. Wage Rigidity

Two sources of variability in wage income are present in the class of contracts set forth in (5): one arises from the stochastic nature of the wage schedule, the other from uncertainty over employment status. That under certain circumstances it is profitable (for both sides) to remove the former is shown in

Lemma 1: Given any feasible variable-wage contract $\delta_1 = \{w(s), n(s)\}$, there exists a feasible fixed-wage contract δ_2 that dominates δ_1, with strict dominance if $u(\cdot)$ is strictly concave.

Proof:[11] Let m satisfy $V(\delta_1, m) = \lambda$, and define $\delta_3 = \{\hat{w}, n(s)\}$ where $\hat{w} = E(wn)/En$. The contract δ_3 produces the same expected labor income for workers and the same expected cost and expected revenue for firms as does δ_1. Note also that

$$\begin{aligned}\Delta \equiv V(\delta_3, m) - V(\delta_1, m) &= \frac{1}{m} En[u(\hat{w}) - u(w)] \\ &\geq (>)\ En(\hat{w} - w)\, u'(\hat{w}), \text{ by concavity (strict concavity),} \\ &= u'(\hat{w})(\hat{w}En - Ewn) = 0.\end{aligned}$$

Since $V(\delta_3, m) \geq \lambda$, there exists a nonnegative (positive if u is strictly concave) constant ε such that the contract $\delta_2 = \{\hat{w} - \varepsilon, n(s)\}$ satisfies $V(\delta_2, m) = V(\delta_1, m)$ and $\pi(\delta_2) \geq \pi(\delta_1)$, QED.

[11] For a similar result in an intertemporal context, see Baily (1974).

A direct corollary of this result is that, in an expected-value sense, enforceable labor contracts dominate labor auctions. Let $w(s)$ be the (parametric) wage schedule associated with an auction market, $f(\cdot)$ be the production function of the typical firm, and $F(\cdot)$ be the inverse of the marginal product function. The outcome of the auction is then equivalent to the contract $\delta_a = \{w(s),\ F[w(s)/p(s)]\}$, which, for some nonnegative ε, is clearly dominated by the contract $\delta = \{Ew(s) - \varepsilon,\ F[w(s)/p(s)]\}$.

Two further points should be emphasized. First, workers who are neutral toward risk are indifferent between a variable-wage contract $\delta_1 = \{w(s),\ n(s)\}$ and a fixed-wage one $\delta_2 = \{Ew(s),\ n(s)\}$. Second, the primary use of lemma 1 is to reduce the complexity of the ensuing mathematics without affecting the ultimate results. A literal interpretation of it would be misplaced, for it is not robust to changes in assumptions about the indivisibility of leisure, the enforceability of contractual bargains, the nature of the random disturbance, and, possibly, the risk of ruin to the firm. With variable hours of work, for instance, wage income $w(1 - l)$ becomes state invariant if preferences are additive; if not, a more complicated function of income and leisure will still be nonstochastic.

Nor should one jump to the conclusion[12] that the real wage schedule is invariant to random disturbances in *aggregate* demand. In fact, theories of speculative labor supply[13] seem to argue against such invariance: when prices are higher than average, workers desire to accumulate assets faster than they normally do. To support a higher flow of savings, they will increase their "propensity" to save and accept a somewhat lower wage in terms of commodities, so that commodity producers have some incentive to hire a larger-than-normal amount of labor services.

In addition, few real-world agreements can prevent workers laid off in a temporarily depressed industry from auctioning off their services to the highest bidder in another industry. What effect these instances of imperfectly enforceable contracts will have on the wage schedule industry is not immediately obvious, but, as a brief argument in Section IV suggests, the outcome will likely be a softening of wage rigidity.

IV. Are Full-Employment Contracts Optimal?

Let $\Phi = \langle \delta_f \rangle$ be the class of all full-employment contracts that are feasible for the typical firm, and imagine that we isolated the dominant member, $\delta_f^* = \{w_f^*,\ m_f^*\}$, of that class. Consider now the class $D = \langle \delta \rangle$

[12] One might be tempted in that direction by the apparent failure of econometric work to find a significant correlation between trend-corrected average real compensation per man-hour (obviously a procyclically biased proxy for the real wage rate) and any price index. Cf. Bodkin (1969), who also reviews earlier efforts in that field.

[13] See Tobin (1947) and Lucas and Rapping (1969).

of all feasible contracts with the same labor force as δ_f^* which are formed from it by reducing employment below m_f^* in at least one state; that is, $\delta = \{w, n(s)\}$ where w is a nonstochastic parameter at least equal to w_f^* and

$$n(s) \leq m_f^* \text{ for all } s, < m_f^* \text{ for some } s. \tag{6}$$

To continue the process of whittling away at the firm's opportunity set, we ask now whether there is an underemployment contract in D which dominates δ_f^*. Note that full-employment agreements possess two desirable features, namely, zero variance in labor income and the lowest wage rate consistent with atomistic competition in the labor market. Against these advantages one must weigh the drawback of a state-invariant employment schedule which does not permit full-employment contracts to vary work force directly with the relative product price, that is, to use a larger amount of labor services in more profitable states of nature.

Every underemployment bargain in class D will pay a wage $w > w_f^*$ to compensate employees for the risk of temporary job loss. For small risks, the *average underemployment premium*, $w - w_f^*$, can be computed from the requirement that all members of D are feasible contracts. Thus, if $\bar{n} \equiv En$,

$$\lambda = (\bar{n}/m_f^*)\, u(w) + (1 - \bar{n}/m_f^*)K \Rightarrow m_f^*(\lambda - K) = \bar{n}[u(w) - K].$$

Note that $u(w_f^*) = \lambda$. Expanding $u(w)$ about w_f^* and neglecting terms of order higher than first, we obtain

$$m_f^*(\lambda - K) = \bar{n}[\lambda + (w - w_f^*)\, u'(w_f^*) - K], \tag{6a}$$

whence

$$(w - w_f^*)\bar{n} = (m_f^* - \bar{n})\, (\lambda - K)/u'(w_f^*). \tag{7}$$

The expected wage bill, $w\bar{n}$, of any δ in D may be thought of as consisting of two parts: a direct expected cost, $w_f^*\bar{n}$, equal to the minimum dollar outlay required to secure the average amount of labor used; and a premium expected cost $(w - w_f^*)\bar{n}$. Expected profit from contract δ is then

$$\pi(\delta) = E(p(s)f[n(s)] - w_f^*\, n(s) - [m_f^* - n(s)](\lambda - K)/u'(w_f^*)). \tag{8}$$

Now let

$$\phi(w_f^*) = (\lambda - K)/u'(w_f^*) = [u(w_f^*) - K]/u'(w_f^*) \tag{9}$$

denote the *marginal* underemployment premium, that is, the increment to premium cost required to compensate the labor force for the loss of one job in any state. The profit contribution of the nth laborer in state s is

$$p(s)\, f'(n) - w_f^* + \phi(w_f^*). \tag{10}$$

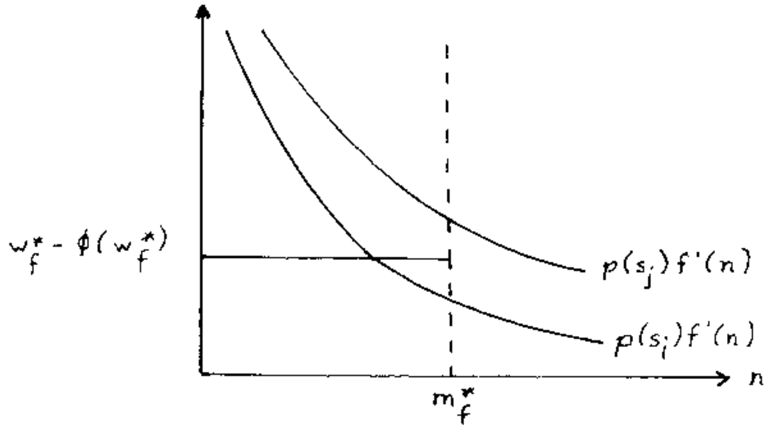

FIG. 1.—Inferiority of full-employment contracts

If there is a state $\hat{s}$ such that

$$p(\hat{s})\, f'(m_f^*) + \phi(w_f^*) < w_f^*, \tag{11}$$

it follows that, relative to δ_f^*, expected profit will be raised if workers are laid off in state $\hat{s}$. *For full-employment contracts to be suboptimal, it suffices, then, that there exist a state of nature in which the fully employed wage rate exceeds the sum of the marginal product value of fully employed labor plus the marginal underemployment premium.* The implication is (fig. 1) that, if s_i is the worst state, full-employment contracts will be suboptimal. It is not clear what type of contract is best when s_j is the lowest state—unless we are certain that (11) is both necessary and sufficient for the inferiority of full-employment agreements.[14] This is examined next.

The Maximum Problem

The fixed-wage feasible contract that firms offer will be the one that generates the highest expected profit. Given any $\delta = \{w, n(s)\}$ with $u(w) > K$ and a positive number m such that

$$\lambda = E[(n/m)u(w) + (1 - n/m)K] \Leftrightarrow m = (\lambda - K)^{-1}[u(w) - K]En,$$

feasibility is equivalent to $n \leq m$, or

$$n(s) \leq (\lambda - K)^{-1}[u(w) - K] \sum q(s)\, n(s) \qquad \forall s. \tag{12}$$

Expected profit is

$$\pi(\delta) = \sum q(s) \left(p(s) f[n(s)] - wn(s)\right), \tag{13}$$

[14] It might be, for instance, that when s_j is the worst state δ_f^* is dominated by an underemployment contract which does not belong to class D.

where the production process uses one single input, labor, and satisfies the usual monotonicity and strict concavity requirements. We assume, further, that

$$f'(n) \to \infty \text{ as } n \to 0,\ f'(n) \to 0 \text{ as } n \to \infty. \tag{14}$$

The decision problem is then to find a *nontrivial* solution [i.e., such that $n(s) > 0$ for some s] to solve the problem

$$(P)\ \max_{\{\delta\}} \pi(\delta)$$

subject to (12) and nonnegativity. Using the one-to-one mapping

$$(w, n) \to (t, n): t = n[u(w) - K],$$

one easily shows

Lemma 2: (P) has at least one nontrivial solution, exactly one if u is strictly concave.

Let $\mu(s)$ be the set of nonnegative multipliers corresponding to the set of inequalities in (12). It follows from lemma 2 and (14) that the unique solution to (P) is completely characterized by the (Kuhn-Tucker) conditions

$$-1 + u'(w) \sum \mu(s)/(\lambda - K) = 0, \tag{15}$$

$$q(s)\{p(s)f'[n(s)] - w + (\lambda - K)^{-1}[u(w) - K] \sum \mu(s)\} = \mu(s) \qquad \forall s, \tag{16}$$

$$n(s) \leq (\lambda - K)^{-1}[u(w) - K] \sum q(s)\, n(s), = \text{if } \mu(s) > 0 \qquad \forall s. \tag{17}$$

Equation (15) confirms lemma 1 and is useful in eliminating $\sum \mu(s)$ from (16). Then

$$q[pf'(n) - z(w)] = \mu \qquad \forall s \tag{16a}$$

where the wage rate net of the marginal underemployment premium,

$$z(w) \equiv w - [u(w) - K]/u'(w), \tag{18}$$

represents the reduction in labor cost accomplished by eliminating one job in any state.

The Inferiority of Full-Employment Contracts: A Necessary and Sufficient Condition

Let h be the inverse function u^{-1}. From feasibility and equation (13), one readily obtains

Lemma 3: $w_f^* = h(\lambda)$; $m_f^* = F[h(\lambda)/Ep]$.

We can now formalize the preceding argument in

Theorem 1: There exists an underemployment contract, δ^*, which dominates δ_f^* if and only if

$$p(s)f'(m_f^*) < z(w_f^*) \text{ for some } s \in S. \tag{19}$$

Proof: Sufficiency. Recall that $\mu(s) \geq 0, \forall s$. If (19) holds, then δ_f^* clearly violates (16a) for some state; neither it nor any other full-employment contract can then be optimal. Necessity. We will use a constructive proof of the contrapositive statement, that is, that

$$p(s)f'(m_f^*) \geq z(w_f^*) \qquad \forall s \in S \tag{20}$$

implies optimality of δ_f^*. Indeed, if (20) holds, then we can define nonnegative multipliers $\mu^*(s)$ such that

$$\mu^*(s) = q(s)[p(s)f'(m_f^*) - z(w_f^*)] \qquad \forall s. \tag{21}$$

Summing both sides of (21) over s, one obtains

$$\sum \mu^*(s) = f'(m_f^*)Ep - z(w_f^*) = (\lambda - K)/u'(w_f^*)$$

by (18) and lemma 3. It follows that the vector $[w_f^*, m_f^*, \mu^*(s)]$ satisfies conditions (15), (16a), and (17) and, therefore, (15), (16), and (17). If (20) is true, then δ_f^* is the optimal contract, QED.

This result is helpful in reducing the issue of underemployment ever being optimal, in an expected value sense, to one of whether or not a given inequality holds. What is it in labor force preferences of consumption versus leisure, in their attitude toward risk, and in material endowments and industry demand characteristics that makes (19) more likely, or less likely, to prevail?

It is useful to limit temporarily the range of attitudes toward risk under consideration by assuming that the *index of relative risk aversion*,

$$r \equiv -(\alpha + w)u''(w)/u'(w), \tag{22}$$

is independent of $\alpha + w$.[15] We can thus concentrate solely on preferences embodied in linear transformations of the following functions:[16]

$$u(w) = (\alpha + w)^{1-r} \qquad \text{if } r < 1 \tag{23a}$$

$$= \log(\alpha + w) \qquad \text{if } r = 1 \tag{23b}$$

$$= -(\alpha + w)^{-(r-1)} \qquad \text{if } r > 1. \tag{23c}$$

I summarize in the next two lemmas some useful bits of information. Both proofs are straightforward.

Lemma 4: Let $k \equiv u^{-1}(K)$ be the wage rate which makes workers indifferent between work and leisure. Then the function $z(w)$ is decreasing

[15] Recall that $u(w) = v(\alpha + w, 0)$, where the constant α is nonlabor income. Arrow (1971, pp. 96–97, 103–4) argues that r is weakly increasing in wealth partly on the basis of empirical findings that the wealth elasticity of money demand is at least 1 (which makes money a "luxury" good). The evidence, however, seems less than overwhelming that demand for cash balances has *significantly* higher than unitary elasticity (Meltzer 1963, pp. 236–38; Laidler 1969, pp. 98–102).

[16] See Pratt (1964, pp. 133–35) for details.

for all $w \geq k$, is invariant to linear transformations of the utility function v, and satisfies $z(k) = k$.

The lemma immediately below is based on the assumption that r is constant.

Lemma 5: The equation $z(w) = 0$ has a unique root, w_0, which satisfies (i) $w_0 = ke - \alpha$ if $r = 1$, $= kr^{1/(r-1)} - \alpha$ if $r \neq 1$, where e is the natural logarithm base; and (ii) w_0 is a decreasing function of r such that $w_0 \to \infty$ as $r \to 0$, $\to -\alpha$ as $r \to \infty$.

These lemmas point out two factors, the workers' opportunities outside the industry and their attitude toward risk, which bear on the optimality of full-employment contracts in an industry. For a large enough value of the index of relative risk aversion, w_0 will inevitably fall short of w_f^* and hence $z(w_f^*) \leq 0$, which rules out (19). Underemployment is more likely to occur, on the other hand, for values of r low enough to place $z(w_f^*)$ in the interval $(0, k]$. Similarly, the higher aggregate demand is (or whatever other exogenous parameter happens to define the economy-wide outlook for these workers), the more attractive working is relative to not working and the larger w_f^* becomes relative to w_0. This lowers the probability that (19) will hold.[17]

Other determinants of the likelihood of layoffs are readily apparent in (19). Layoffs should be more frequent in industries with relatively volatile (high variance in s) or relatively inelastic (p highly responsive to s) demand schedules, in markets with relatively little competition, and in cases of relatively high unemployment compensation (which boosts K and thereby depresses the marginal underemployment premium).[18]

Divisible Leisure

If one were interested in what statements a theory of labor contracts might make about, say, the behavior of hours worked over the cycle or about seniority claims on overtime, the obvious extension of the simple contract form is

$$\delta = \{w(s, t), x(s, t), n(s, t)\},$$

where $x(s, t) = 1 - l(s, t)$ is the amount of hours worked by each laborer if state s occurs in period t. As one might expect from Section III, such

[17] Since the constant k in lemmas 4 and 5 depends on α, it is not obvious in this model how the size of nonhuman wealth affects unemployment. For a *fixed* economy-wide full-employment wage rate, both working and not working become more attractive as nonlabor income rises. If, on the other hand, wealthier workers were able to hold out—as they might in a search-oriented model—for better opportunities than could their poorer colleagues, nonhuman wealth and the probability of accepting a uniform contract offer might be negatively correlated (see Danforth 1974).

[18] For additional factors affecting layoffs, see Gordon (1974).

augmented contracts will soften wage rigidity somewhat but affect little else. Of particular interest for this paper is whether these agreements will react to a decline in product demand by reducing, primarily, hours worked rather than employment and thereby prevent (19) from holding.

One reason we do not observe arbitrarily short work schedules[19] has to do with the well-known tendency of people to prefer lumpy leisure streams over entirely smooth ones, for example, long weekends, paid vacations, etc. Suppose, for instance, that labor preferences over alternative consumption and leisure streams are represented by monotone transformations of the additive function

$$v(c, l) = E\{\sum_{t=0}^{T} \beta^t[u(c) + k(l)]\}. \tag{24}$$

where the expectation is taken with respect to the joint distribution $q(s_0, s_1, \ldots, s_T)$ of states of nature over the (presumed exogenous) duration of the contract. Then the tastes for lumpy leisure streams would be reflected in the requirement that $k(l)$ be concave over some interval $(0, l_0)$ and convex over $(l_0, 1)$. And work schedules which consume nearly all or nearly none of the leisure available to workers will bear prohibitively high "overtime" or "undertime" costs. The latter type of premium simply reflects the fact that it takes a very high hourly wage to persuade people that they must abandon the comforts of home for 1 hour's work at the factory.

Is Contractual Unemployment Involuntary?

The rest of the conclusions drawn from theorem 1 are unexceptional, perhaps to the point of fomenting suspicion that they could be arrived at via the familiar auction model with some restrictions on labor mobility. Indeed, the perfect enforceability of contracts, an assumption uncomfortably close to indenture, does guarantee that whoever is laid off when (19) prevails will be unable to seek employment outside the industry. Furthermore, if interindustry labor mobility is low because of, say, moving costs or industry-specific skills, some (voluntary) unemployment will occur in states for which a condition reminiscent of (19) holds, that is,

$$p(s) f'(\mu) < k, \tag{25}$$

where μ = labor stock per firm.

On reflection, perfect bilateral enforceability of contracts does not seem to be as stringent a restriction as its name would imply. Suppose, for instance, that such agreements are one-sided bargains enforceable on the

19 Setup costs in production is another obvious cause, one that has received much formal attention in inventory theory.

employer but not on the employee.[20] Persons whose contracted services are temporarily not in demand would be the ones most likely to prefer a job outside the industry in question over their current status of pure leisure consumption. Such temporary jobs, probably offered in auction, will materialize only if there exist industries with demand sufficiently strong to ensure full employment to their permanent labor force and to justify hiring *additional* help (filling "vacancies") at wages no smaller than k.

Idled workers will search actively if and only if temporary jobs outside their industry are available at wage rates sufficient to compensate them for the loss of leisure. In times of relatively slack aggregate demand, temporary vacancies will tend to fall relative to layoffs; hence, the probability of landing any temporary job (alternatively, the market-clearing auction wage) will fall and search will be discouraged. The converse will be true in periods of above-average aggregate demand: the auction wage will be high compared with the minimum working wage k, and searchers may well be able to secure employment at wages in excess of what prevails in the contractual labor market. This implies that firms which do not wish to have their production plans upset by a high quit rate cannot afford to maintain a rigid wage, at least not upward, against protracted fluctuations in aggregate demand.

V. Optimum Contracts—a Complete Characterization

Thus far we have derived a condition under which full-employment contracts are suboptimal. Recall the assumption that all contracts are strictly enforceable and have a parametrically given equilibrium value, λ. Without further loss of generality, suppose that the distribution of s is uniform in the interval (0, 1). The optimal contract is then completely characterized by[21]

Theorem 2: If (19) holds for all $s \leq \hat{s}$ and some $\lambda \in (K, \infty)$, then there exist two continuous decreasing functions $M(\lambda)$ and $Y(\lambda)$ and a continuous increasing function $T(\lambda)$, all three unique and such that

(i) $w_f^* < T(\lambda) < w_0 \quad \forall \lambda \in (K, \infty)$;

(ii) $M(\lambda) \to \infty$ as $\lambda \to K$, $M(\lambda) \to 0$ as $\lambda \to \infty$;

(iii) $M(\lambda) > m_f^* \quad \forall \lambda \in (K, \infty)$;

(iv) $Y(\lambda) \to 1$ as $\lambda \to K$, $Y(\lambda) \to 0$ as $\lambda \to \infty$;

(v) $p[Y(\lambda)]f'[M(\lambda)] = z[T(\lambda)] \quad \forall \lambda \in (K, \infty)$.

[20] Casual empiricism reinforces this belief to the extent that it is easier, or less costly, to secure information on the reputation and past personnel policies of firms than on the employment record of not very highly paid employees.

[21] The proof, which essentially involves characterizing the solution of two simultaneous nonlinear equations in two unknowns, is available on request from the author.

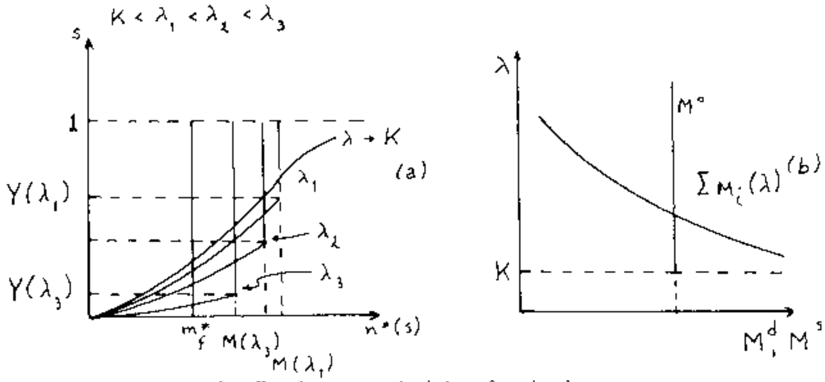

FIG. 2.—Employment schedules of optimal contracts

Furthermore, the optimal contract $\delta^* = \{w^*, n^*(s)\}$ satisfies

(vi) $Epf'(n^*) = w^* - \psi(w^*) < w^*$;

(vii) $w^* = T(\lambda)$;

(viii) $\psi(w^*) = [u(w^*) - \lambda]/u'(w^*) > 0$;

(ix) $n^*(s) = F[z(w^*)/p(s)]$ for $s < Y(\lambda)$

$= M(\lambda)$ for $s > Y(\lambda)$;

(x) $n^*(s)$ is continuous, nondecreasing in s, increasing in λ for $s < Y(\lambda)$, and decreasing in λ for $s > Y(\lambda)$; and

(xi) $n^*(s) = m_f^*$ at some $s \equiv s^* < \min\{\bar{s}, Y(\lambda)\}$.

This theorem argues that the wage associated with the underemployment contract exceeds its full-employment counterpart, and so does work force size. The first of these two departures from full-employment norms is necessary to compensate laborers for a positive probability of unemployment; the second one allows employers to shift output from relatively unprofitable to relatively profitable states of nature.

As λ rises relative to both the product price schedule, $p(s)$, and the value of leisure, it becomes more "expensive" for the firm to maintain a risky, that is, steeply sloped, employment schedule. Thus, the underemployment and full-employment portions of $n^*(s)$ shift to the right and left, respectively (fig. 2*a*), the critical state $Y(\lambda)$ declines in value, and $n^*(s)$ comes closer to a full-employment schedule.

Workers in active employment are paid a wage rate[22] which exceeds the expected value of labor's marginal product by a marginal underemployment premium $\psi(w^*)$. A comparison with equation (9) shows that

[22] There seems to be no relationship between the full-employment wage rate and the expected marginal product of labor under the optimal contract.

ψ is a smaller function than ϕ, that is,

$$\phi(w) - \psi(w) = \lambda - K > 0 \text{ for all } w \geq k. \tag{26}$$

The difference arises from ϕ and ψ being the marginal underemployment premium in the class D of feasible contracts with fixed labor pool m_f^* and in the class F of all feasible contracts, respectively. If employment below m_f^* is called for in some states, class D must inevitably present greater unemployment risks than class F: the option of varying the size of the labor force is available only in the latter.

For fixed tastes, technology, and population, the equilibrium value of contracts is determined, in the usual manner (fig. 2*b*), by the interaction of a fixed supply $M^s = M^0$ of workers with economy-wide demand, $M^d = \sum_i M_i(\lambda)$, for "work force," where i indexes industries.

Contracts versus Auction[23]

We return now to the issue of whether the employment (and output) pattern in a labor contracts economy is identical to, or varies systematically from, what would prevail if labor services were sold in instantaneous auction. With unlimited labor mobility, the latter type of market organization is Pareto efficient and, hence, must generate at least as high a volume of employment and output as a market for contractual bargains.

A less vacuous comparison would recognize that low mobility is one of the primary reasons long-term attachments arise between workers and firms. Reduced to its barest essentials, the issue is: given a labor stock of μ workers (in per firm terms) without job opportunities outside the industry, how does the outcome of an auction market compare with the terms of the optimal contract?

Let $w_a(s)$, $n^s(s)$, $n^d(s)$, and $n_a(s)$ be the wage, labor supply, labor demand, and employment schedules, respectively, in the auction market. Define s^0 from

$$p(s^0)f'(\mu) = k \tag{26a}$$

and note that $k = z(k)$. Next let the function $g(w, m)$ solve

$$p(g)f'(m) = z(w). \tag{27}$$

Now one can show[24]

Lemma 6: The function $g(w, m)$ is unique, continuously differentiable, decreasing in w, increasing in m, and bounded above by 1 and below by 0, and is such that (i) $s^0 = g(k, \mu)$ solves (26a); and (ii) $g(k, \mu) > Y(\lambda)$, where $Y(\lambda)$ is defined in theorem 2.

[23] I am indebted to Milton Friedman for provoking this section.
[24] N. 21 applies here, too.

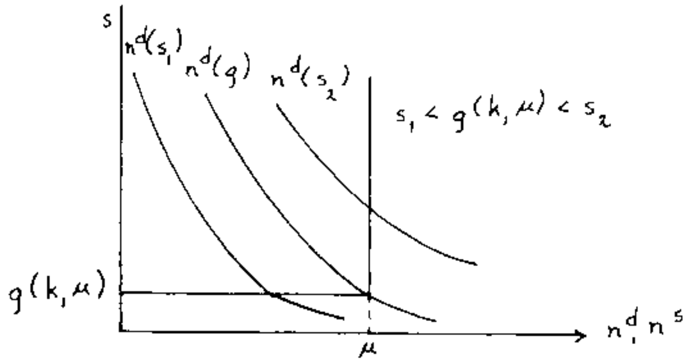

FIG. 3.—Auction market equilibrium

Now since

$$n^d(s) = F[w(s)/p(s)]; \; n^s(s) = \mu \text{ if } w_a(s) > k, \qquad (28)$$
$$= 0 \text{ if } w_a(s) < k,$$

it follows from lemma 6 that

$$n_a(s) = F[k/p(s)], \; w_a(s) = k \text{ if } s < g(k, \mu) \qquad (29)$$
$$= \mu \qquad , \qquad = p(s)f'(\mu) \text{ if } s > g(k, \mu).$$

These relations are graphed in figure 3.

In the contractual market, assume that a full-employment contract paying the lowest possible wage is profitable on average, that is,

$$f'(\mu)\, Ep > k. \qquad (30)$$

The optimal agreement will be of the form $\delta_f^* = \{f'(\mu)Ep, \mu\}$ if

$$p(s)f'(\mu) \geq z[f'(\mu)Ep] \qquad \forall s \qquad (31)$$

holds; otherwise, theorem 2 is in force, with the additional proviso that

$$M(\lambda) = \mu. \qquad (32)$$

Let w^*, $n^*(s)$, and $\bar{w}_c$ be the wage rate, employment schedule, and expected wage income under the optimal contract and let $\bar{w}_a$ be the expected wage income in an auction market. Note that

$$\bar{w}_a = z(k)g(k, \mu) + \int_{g(k,\mu)}^{1} f'(\mu)p(s)ds. \qquad (33)$$

Now, if (31) is true, then $\bar{w}_c = w_f^* = f'(\mu)Ep$ and $n^*(s) = \mu$ for all s. Hence,

$$\bar{w}_c - \bar{w}_a = \int_0^g [f'(\mu)p(s) - z(k)]ds < 0 \qquad (34a)$$

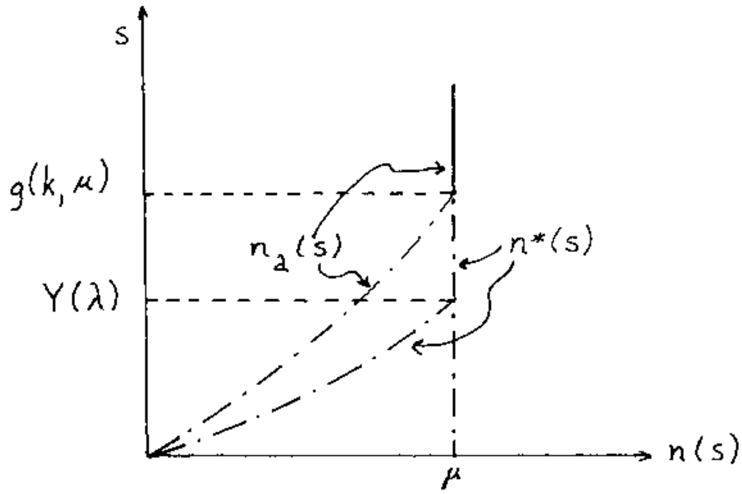

FIG. 4.—Comparative employment

and, from (29),

$$n^*(s) \geq n_a(s), > \text{ if } s < g(k, \mu). \tag{35}$$

If, on the other hand, (31) does not hold, then we know that $z(w^*) < k$ and $w^* \in (w_f^*, w_0)$; hence, part (ix) of theorem 2 suggests that (35) is equally true in this case also. Note, further, that in this case again

$$\bar{w}_c = w^* En^*/\mu = w^*(\lambda - K)/[u(w^*) - K],$$

which is readily shown to be a decreasing function of w^* for all w^* in the relevant range (k, w_0) and all $\lambda > K$. Hence,

$$\bar{w}_c < w_f^*(\lambda - K)/[u(w_f^*) - K] = w_f^* = f'(\mu)Ep < \bar{w}_a. \tag{34b}$$

Observe also from theorem 2(v) and lemma 6 that

$$Y(\lambda) \equiv g[T(\lambda), M(\lambda)] = g(w^*, \mu) < g(k,\mu).$$

This completes the proof of

Theorem 3: (i) $\bar{w}_c < \bar{w}_a$; and (ii) $n_a(s) \leq n^*(s)$ $\forall s$, $<$ for $s < g(k, \mu)$; where (iii) $g(k, \mu) > Y(\lambda)$.

As one might expect, labor markets generate riskier employment schedules (fig. 4) and larger expectations of wage income when they are organized as auctions rather than as contract exchanges.

VI. Summary and Conclusions

This paper examines a simple industry with demand uncertainty which motivates risk-neutral firms to act both as employers and as insurers of homogeneous, risk-averse laborers. The resulting contractual agreements turn out, in their simplest form, to be more likely to specify full employment the more of the following conditions prevail: small variability in product price, above-average economy-wide labor demand, highly risk-averse workers, small unemployment compensation, and highly competitive product market. Otherwise, it may be optimal to lay off, by random choice, part of the work force during low states of demand.

In such "underemployment" contracts it is profitable, under certain assumptions, to eliminate wage fluctuations, but not employment status, as a source contributing to wage income variability. If the labor force is industry specific, contractual organization of the labor market will result in the volume of employment being no lower in any state, and higher in some states, than that associated with a labor auction.

The employment path generated in a contractual labor market is not much different from the one which prevails in a spot auction market; because of rigid wages, however, reductions in employment appear as layoffs rather than as voluntary withdrawals of labor services.

Some extensions of the work reported in Sections II–V are worth contemplating. The first involves dropping the assumption of perfect homogeneity of the labor force and finding out what sort of wage distributions, layoff provisions, and participation rates are consistent with labor market equilibrium when different contracts are offered to persons of varying technical skills, tastes, asset positions, and home production opportunities. A related paper of mine (Azariadis 1975) examines the incidence of unemployment in a technology with two asymmetrically substitutable skill grades of labor. That paper finds that the trade-offs between wage income and the expected rate of unemployment embodied in optimal contracts clearly favor skilled workers over common laborers.

Another extension, passingly discussed in Section IV, is relaxing the restriction of contract enforceability, at least on employees. In a model with at least two industries whose product price ratio varies according to a well-defined stochastic process, unilaterally enforceable contracts may lead to a natural definition of (temporary) vacancies and, in all likelihood, to a softening of wage rigidity.

Perhaps the most interesting area of application for the class of models treated in this paper arises when the assumption of exogeneity in the price level is dropped. Perceived disturbances of the price level relative either to its average long-term value or to the price of commodities in informationally segmented markets are at the heart of modern treatments of the Phillips curve.[25] Whether we can model price movements of this sort to learn something about wage rigidity in the large and, more generally, optimal escalator clauses in money-wage contracts seems to me an issue worth pursuing.

[25] See Friedman (1968), Phelps (1968), and Mortensen (1970) on the role of movements in contemporaneous price ratios and Lucas and Rapping (1969) and Lucas (1972) on the role of movements in intertemporal ones.

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