Economics 202b; Fall 2000; Final Exam

1. Monetary Policy: Suppose that the level of real output in a discrete-time model of the economy is given by the IS equation:

(1) $y_t = -\beta r_{t-1} + \lambda y_{t-1} + \varepsilon_t$

where y is the log difference between actual output and potential output, r is the real interest rate, β and λ are parameters, and ε is an independent and identically distributed shock to the IS curve. Suppose further that the economy has adaptive expectations of inflation, so that the rate of inflation is given by the Phillips curve equation:

(2) $\pi_t = \pi_{t-1} + \alpha y_{t-1} + \eta_t$

where π is the inflation rate, α is a parameter, and η is an independent and identically distributed shock to the Phillips curve. Suppose further that the central bank chooses the interest rate according to a rule:

(3) $r_t = \theta y_t + \phi \pi_t$

in order to minimize the objective function:

(4) $E\left\{(1-\gamma)\pi^2 + \gamma y^2\right\}$

where γ is a parameter that governs the central bank's relative aversion to output variability.

a. Solve for π_t, this year's inflation rate, as a function of inflation and output two years ago and of the ε and η shocks since.
b. Suppose that γ=0. What do θ and φ need to be in order to minimize the central bank's objective function?
c. Suppose that γ =1. What do θ and φ need to be in order to minimize the central bank's objective function?
d. What can you say about the values of θ and φ if g is between zero and one?
e. Explain—in about one page in your exam book—why Alan Greenspan or some other senior official at a central bank should find the above mathematical exercise very interesting. What conclusions about monetary policy can be drawn from it?

2. Fiscal Policy: Suppose that there is a continuous-time small open economy populated by identical agents, each of whom has a constant probability of death per unit time of p.

While each person is alive he or she works, and is paid at a normalized after tax wage rate of 1-T. Everyone expects the tax rate to be constant, and the same in the future as in the past.

Agents are free to borrow and lend at the same market equilibrium interest rate r that is the yield on capital. Because the economy is a small open economy, the interest rate r is fixed and unchanging. Moreover, when agents borrow and lend, they insure against death. For each unit of their wealth, they purchase a particular anti-life-insurance policy from an insurance company—a policy called a *tontine*. The insurance company pays the individual p per unit of time per unit of wealth as the individual is alive, and in exchange receives title to their wealth at the individual's death. (This is an actuarially fair policy).

Thus if they lend one unit of principal, they receive interest at a rate r per unit of time from the borrower and tontines at a rate p per period of time from the insurance company (and their loan is transferred to the insurance company upon death). If they borrow, they pay interest at a rate r per unit of time to the lender, and also pay an insurance premium at a rate p per unit of time to the insurance company that commits to cover their debt should they die before repayment.

Individuals maximize a logarithmic utility function. An individual born at time s looking forward as of time t maximizes:

$$(1) \qquad \left\{ \int_{v=t}^{\infty} \log(c_v^s) e^{p(t-v)} e^{\theta(t-v)} dv \right\}$$

Where c_t^s is the consumption at time t of an agent born at time s, where θ is the time discount parameter, and where the extra factor $e^{p(t-v)}$ arises from the fact that some members of your cohort will die between now and the future. That people born when you were born will have a certain consumption level at some time t far in the future will be of no interest to you if you die before time t arrives.

a. What is the intertemporal Euler equation for consumption of an agent born at time s and still alive at time t? What is dc_t/dt?

If at time t an agent born at time s has non-human wealth—property—of amount w_t^s, the dynamic budget constraint that he or she faces is:

(2) $$\frac{dw_t^s}{dt} = [r + p]w_t^s + 1 - T_t - c_t$$

which merely states that property income plus labor income minus consumption equals the change in (non-human) wealth.

b. From the Euler equation and the budget constraint, either derive or argue heuristically that the economically-sensible consumption function of such an agent is:

$$c_t^s = (\theta + p)(w_t^s + h_t)$$

Where:

$$h_t = \int_{v=t}^{\infty} 1 \times e^{-(r+p)(v-t)} dv = \frac{1-T}{r+p}$$

is the human wealth—the appropriately-discounted future labor income—of an agent alive at time t, and w_t^s is the agent's non-human wealth.

Now aggregate up. From here on work with economy-wide totals of consumption C, income Y, and wealth W rather than individual amounts. Assume that at every instant a large cohort, normalized to be of size (or measure) p is born. The size of a cohort born at time zero still alive at time t is thus equal to pe^{-pt}. And the size of the whole population at any time t is:

(3) $$\int_{s=-\infty}^{t} pe^{-p(t-s)} ds = 1$$

Thus the total population alive is unchanging and is equal to one, and the total level of income Y in the economy is equal to one as well.

c. What is aggregate consumption C in this economy as a function of total economy-wide wealth W?
d. What is dW/dt as a function of total economy-wide wealth W, and the parameters r, p, and θ? (Be careful! Consider that the component p x w of each individual's income is not an addition to aggregate wealth, but only a transfer from those who die to those who remain alive.)
e. What happens to the level of consumption in this economy if the government conducts a fiscal policy—that is, transfers a total amount of wealth G in the form of

bonds to the currently-alive, and increases taxes by rG in order to pay the interest on these bonds?

f. What does this model tell us about the debate over Ricardian equivalence? (No more than one page, please!)

3. Exchange Rate Crises: Suppose that the demand for money balances in a small open economy is given by:

$$m_t - e_t = -\eta \times \frac{de_t}{dt}$$

where m is the (log) money stock, e is the (log) exchange rate, and η is a parameter. Suppose that the government does not have power to print banknotes: it must work through the central bank, which conducts open-market operations—sales of bonds for banknotes, or purchases of bonds for banknotes—and foreign-exchange interventions—sales of domestic bonds for foreign bonds, or purchases of domestic bonds for foreign bonds. For simplicity's sake, suppose that the interest the central bank earns on its bond holdings is just sufficient to cover the costs of running the central bank.

a. Explain why, with this institutional setup for the central bank, before any speculative attack the money stock is equal to:

$$M_t = B_t + E^* F_t$$

where B is the central bank's net holdings of domestic bonds, E* is the (fixed) level of the exchange rate, and F is the central bank's net holdings of foreign exchange reserves.

b. Suppose that the central bank keeps the log exchange rate fixed at e*. What path must the money stock follow in order to maintain this exchange rate target?
c. Suppose that government fiscal policy is such as to require that the central bank increase its holdings of domestic bonds at a proportional rate θ as long as it maintains its fixed exchange rate. What is the time path of the central bank's holdings of foreign exchange reserves as long as the fixed exchange rate target is maintained?

When a speculative attack takes place the government responds by selling its foreign-currency reserves for domestic bonds, and then turning around and selling the excess domestic bonds for cash in order to keep it on its domestic debt accumulation path. Thus a speculative attack does not cause a discontinuous change in B, but it does cause a discontinuous change in M.

Suppose that after a speculative attack government fiscal policy also changes—so that after the shock of the attack central bank holdings of domestic bonds grow not at a proportional rate θ but at some alternative rate φ. And suppose that everyone in this economy has *perfect foresight*.

d. Suppose $\varphi=\theta$ and both are greater than zero. Will this economy suffer a speculative attack on its exchange rate? If so, when?
e. Suppose θ is greater than zero, but that $\varphi=0$. Will this economy suffer a speculative attack on its exchange rate? Why or why not? What happens after the central bank's foreign exchange reserves are exhausted?
f. Suppose $\theta=0$, but that φ is greater than zero. Can this economy suffer a speculative attack? If so, under what circumstances can it suffer a speculative attack? Will it suffer one?
g. Suppose that you are in charge of writing a memo for a non-economist who has just become head of your country's central bank and wants to know what he or she should do to guard against a speculative attack on the currency. How does your analysis above inform what you write? What do you say? (One page only, please!)