Economics 202b; Fall 2000; Final Exam Answers

1. Monetary Policy: Suppose that the level of real output in a discrete-time model of the economy is given by the IS equation:

(1) $y_t = -\beta r_{t-1} + \lambda y_{t-1} + \varepsilon_t$

where y is the log difference between actual output and potential output, r is the real interest rate, β and λ are parameters, and ε is an independent and identically distributed shock to the IS curve. Suppose further that the economy has adaptive expectations of inflation, so that the rate of inflation is given by the Phillips curve equation:

(2) $\pi_t = \pi_{t-1} + \alpha y_{t-1} + \eta_t$

where π is the inflation rate, α is a parameter, and η is an independent and identically distributed shock to the Phillips curve. Suppose further that the central bank chooses the interest rate according to a rule:

(3) $r_t = \theta y_t + \phi \pi_t$

in order to minimize the objective function:

(4) $E\{(1-\gamma)\pi^2 + \gamma y^2\}$

where γ is a parameter that governs the central bank's relative aversion to output variability.

This problem is taken directly from Laurence Ball (1999), "Efficient Rules for Monetary Policy" ***International Finance*****, 2:1, pp. 63-83, on the reading list for the last full week in October, covered in lecture on October 26 and in section on October 27.**

a. Solve for π_t, this year's inflation rate, as a function of inflation and output two years ago and of the ε and η shocks since.

Part a is straightfoward. Begin by substituting equation (1) for y_{t-1} in equation (2), and substitute (2) for π_{t-1} in (2) as well:

$$\pi_t = (\pi_{t-2} + \alpha y_{t-2} + \eta_{t-1}) + \alpha(-\beta r_{t-2} + \lambda y_{t-2} + \varepsilon_{t-1}) + \eta_t$$

Then substitute (3), the rule that the central bank follows in setting the interest rate r, into the equation above:

$$\pi_t = (\pi_{t-2} + \alpha y_{t-2} + \eta_{t-1}) + \alpha(-\beta\theta y_{t-2} - \beta\phi\pi_{t-2} + \lambda y_{t-2} + \varepsilon_{t-1}) + \eta_t$$

And regroup:

$$\pi_t = (1 - \alpha\beta\phi)\pi_{t-2} + \alpha(1 + \lambda - \beta\theta)y_{t-2} + \eta_t + \eta_{t-1} + \alpha\varepsilon_{t-1}$$

b. Suppose that γ=0. What do θ and φ need to be in order to minimize the central bank's objective function?

If γ=0, then the objective function is simply to minimize the expected value of inflation squared. Since at time t-2 we don't know what the future values of η or ε will be, we can write π_t as:

$$\pi_t = E_{t-2}(\pi_t) + \{\eta_t + \eta_{t-1} + \alpha\varepsilon_{t-1}\}$$

where

$$E_{t-2}(\pi_t) = (1 - \alpha\beta\phi)\pi_{t-2} + \alpha(1 + \lambda - \beta\theta)y_{t-2}$$

$\{\eta_t + \eta_{t-1} + \alpha\varepsilon_{t-1}\}$ cannot be affected by government policy: they do not depend on either the value of θ or the value of ϕ in the government's monetary policy rule. But $E_{t-2}(\pi_t)$ can be and is affected by the government's policy rule. As Ball points out, the way to minimize the variability of inflation about zero is to do everything you can to neutralize and offset all the factors affecting inflation that you know about: set your interest rate policy to make expected inflation equal to zero.

If

$$\phi = \frac{1}{\alpha\beta} \quad and \quad \theta = \frac{1+\lambda}{\beta}$$

then the coefficients on past output and past inflation in the answer to (a) are both zero:

$$E_{t-2}(\pi_t) = 0$$

That means that:

$$E(\pi_t)^2 = E(\eta_t)^2 + E(\eta_{t-1})^2 + \alpha^2 E(\varepsilon_{t-1})^2$$

Inflation depends only on shocks that happen after time t-2—thus it depends on things that government policy cannot compensate for. And that is the best you can do to minimize inflation.

c. Suppose that γ =1. What do θ and φ need to be in order to minimize the central bank's objective function?

The central bank cares only about minimizing the output gap squared. The output gap is equal to:

$$y_t = -\beta r_{t-1} + \lambda y_{t-1} + \varepsilon_t$$

Substitute in the central bank's policy rule:

$$y_t = (\lambda - \beta\theta) y_{t-1} - \beta\phi\pi_{t-1} + \varepsilon_t$$

If we set:

$$\phi = 0 \;\; and \;\; \theta = \frac{\lambda}{\beta}$$

Then y_t depends only on shocks that we cannot know and counter as of t-1, and that is the best we can do to minimize the square of the output gap.

d. What can you say about the values of θ and φ if γ is between zero and one?

We can say that the values of φ and θ are somewhere in between the values for the two extreme cases. We can say that the central bank reacts more strongly to movements in the output gap than when it is concerned only with minimizing output variance, and less strongly to movements in the output gap than when it is concerned only with inflation variance. We can say that the central bank should not react at all to inflation if it does not care about inflation—but it should react to output as well as to inflation even if it does not care about output.

e. Explain—in about one page in your exam book—why Alan Greenspan or some other senior official at a central bank should find the above mathematical exercise very interesting. What conclusions about monetary policy can be drawn from it?

This model shows that in a simple adaptive-expectations Phillips curve context a monetary policy of "leaning against the wind"—raising interest rates when either output rises above potential output or inflation rises, and lowering them when either falls—can be an effective stabilization policy. The amount of leaning that should be done depends on the parameters of the economy—the more persistent are fluctuations in the output gap (the higher is λ), the stronger should be the central bank's reaction to movements in output. The more powerful are interest rates at affecting output (the higher is β), the smaller should be the central bank's changes in interest rates. The larger is the effect of an output gap on inflation (the larger is α), the smaller should be the central bank's reaction to inflation. In addition, the central bank should move more aggressively in response to shifts in either output or inflation the stronger is its focus on stabilizing inflation.

2. Fiscal Policy: Suppose that there is a continuous-time small open economy populated by identical agents, each of whom has a constant probability of death per unit time of p.

While each person is alive he or she works, and is paid at a normalized after tax wage rate of 1-T. Everyone expects the tax rate to be constant, and the same in the future as in the past.

Agents are free to borrow and lend at the same market equilibrium interest rate r that is the yield on capital. Because the economy is a small open economy, the interest rate r is fixed and unchanging. Moreover, when agents borrow and lend, they insure against death. For each unit of their wealth, they purchase a particular anti-life-insurance policy from an insurance company—a policy called a *tontine*. The insurance company pays the individual p per unit of time per unit of wealth as the individual is alive, and in exchange receives title to their wealth at the individual's death. (This is an actuarially fair policy).

Thus if they lend one unit of principal, they receive interest at a rate r per unit of time from the borrower and tontines at a rate p per period of time from the insurance company (and their loan is transferred to the insurance company upon death). If they borrow, they pay interest at a rate r per unit of time to the lender, and also pay an insurance premium at a rate p per unit of time to the insurance company that commits to cover their debt should they die before repayment.

Individuals maximize a logarithmic utility function. An individual born at time s looking forward as of time t maximizes:

(1) $$\left\{ \int_{v=t}^{\infty} \log(c_v^s) e^{p(t-v)} e^{\theta(t-v)} dv \right\}$$

Where c_t^s is the consumption at time t of an agent born at time s, where θ is the time discount parameter, and where the extra factor $e^{p(t-v)}$ arises from the fact that some members of your cohort will die between now and the future. That people born when you were born will have a certain consumption level at some time t far in the future will be of no interest to you if you die before time t arrives.

This problem comes from a *Journal of Political Economy* article by Olivier Blanchard, "Debt, Deficits, and Finite Horizons." Its point is to show exactly how the possibility that one might not be around to pay taxes in the future but one is definitely around to consume in the present breaks Ricardian equivalence.

a. What is the intertemporal Euler equation for consumption of an agent born at time s and still alive at time t? What is dc_t/dt?

If you postpone a small amount of consumption Δc for a small amount of time Δt, you will be able to consume more (because you will have been earning interest at rate r+p on your savings), but time preference means that you discount that higher consumption at a rate θ and the risk of death means that you discount that higher consumption at an additional rate p. Because you have log utility, the marginal utility of each extra unit of consumption at t and at t+Δt is inversely proportional to the level of consumption at those dates.

Thus equating (private) marginal rates of substitution to marginal utilities of consumption:

$$e^{-(r+p)\Delta t} = e^{-(p+\theta)\Delta t} \times \frac{c_t}{c_{t+\Delta t}}$$

Taking logs:

$$-(r+p)\Delta t = -(p+\theta)\Delta t - \left(\ln(c_{t+\Delta t}) - \ln(c_t)\right)$$

Dividing by Δt:

$$-(r+p) = -(p+\theta) - \left(\frac{\ln(c_{t+\Delta t}) - \ln(c_t)}{\Delta t}\right)$$

And then taking limits as Δt goes to zero...

$$(r+p) = (p+\theta) + \frac{d}{dt}\ln(c_t)$$

Rearrange as:

$$\frac{1}{c_t}\frac{dc_t}{dt} = r - \theta$$

That is the growth rate of consumption.

If at time t an agent born at time s has non-human wealth—property—of amount w_t^s, the dynamic budget constraint that he or she faces is:

(2) $$\frac{dw_t^s}{dt} = [r+p]w_t^s + 1 - T_t - c_t$$

which merely states that property income plus labor income minus consumption equals the change in (non-human) wealth.

b. From the Euler equation and the budget constraint, either derive or argue heuristically that the economically-sensible consumption function of such an agent is:

$$c_t^s = (\theta + p)(w_t^s + h_t)$$

Where:

$$h_t = \int_{v=t}^{\infty} (1-T) \times e^{-(r+p)(v-t)} dv = \frac{1-T}{r+p}$$

is the human wealth—the appropriately-discounted future labor income—of an agent alive at time t, and w_t^s is the agent's non-human wealth.

Heuristically, whenever you have log utility the optimal consumption path is to at every moment consume a share of your total wealth equal to your rate of time discount. Since the rate of return on your total wealth (human and non-human) is r+p and your rate of time discount is θ+p, that means that if you consume a proportional rate θ+p out of your total wealth then your total wealth and total consumption will be growing at rate r-θ—which is what the Euler equation requires. This consumption rule also satisfies the budget constraint, because it means neither that you have negative total wealth at the end of time, nor does your wealth at the end of time have any positive present value today...

Now aggregate up. From here on work with economy-wide totals of consumption C, income Y, and wealth W rather than individual amounts. Assume that at every instant a large cohort, normalized to be of size (or measure) p is born. The size of a cohort born at time zero still alive at time t is thus equal to pe^{-pt}. And the size of the whole population at any time t is:

(3) $$\int_{s=-\infty}^{t} pe^{-p(t-s)} ds = 1$$

Thus the total population alive is unchanging and is equal to one, and the total level of income Y in the economy is equal to one as well.

c. What is aggregate consumption C in this economy as a function of total economy-wide wealth W?

Since consumption is the same fraction of wealth for every individual, simply add up total wealth for everyone in the economy, multiply by the propensity to consume θ+p, and you have the economy's consumption function:

$$C_t = (\theta + p)(W_t + H_t) = (\theta + p)\left(W_t + \frac{1-T}{r+p}\right)$$

where the last term substitutes in the value of human wealth as defined above.

d. What is dW/dt as a function of total economy-wide wealth W, and the parameters r, p, and θ? (Be careful! Consider that the component p x w of each individual's income is not an addition to aggregate wealth, but only a transfer from those who die to those who remain alive.)

The agents in the economy have two sources of income: the returns on their non-human wealth W (remember! The p-term is a transfer!) equal to rW, and their after tax other income (1-T). The change in wealth will be the difference between income and consumption:

$$\frac{dW_t}{dt} = rW_t + (1-T) - C_t$$

Substituting in the consumption function, we see that total wealth evolves according to:

$$\frac{dW_t}{dt} = rW_t + 1 - T - (\theta + p)\left(W_t + \frac{1-T}{r+p}\right)$$

$$\frac{dW_t}{dt} = (r - (\theta + p))W_t + \left(\frac{r-\theta}{r+p}\right)(1-T)$$

With θ+p greater than r, wealth converges to a steady-state value of:

$$W_t = \frac{(r-\theta)(1-T)}{(\theta + p - r)(r+p)}$$

And consumption converges to a steady-state value of:

$$C = (\theta + p)(W + H) = (\theta + p)\left(\frac{(r-\theta)(1-T)}{(\theta + p - r)(r+p)} + \frac{1-T}{(r+p)}\right)$$

$$C = \frac{p(\theta + p)(1-T)}{(\theta + p - r)(r+p)}$$

e. What happens to the level of consumption in this economy if the government conducts a fiscal policy—that is, transfers a total amount of wealth G in the form of bonds to the currently-alive, and increases taxes by rG in order to pay the interest on these bonds?

Non-human wealth W rises by G, the amount of bonds transferred. Human wealth falls by G(r/(r+p))—the increased taxes rG are discounted at the same rate r+p as wage income, the p emerging because you think that there is a chance you won't be around to pay the taxes. Consumption immediately rises by (θ + p)G(p/(r+p)).

If we call the new wealth W' = W + G, and the new tax rate T' = T + rG, then our same law of motion for wealth holds:

$$\frac{dW_t}{dt} = (r - \theta - p)\left(W_t - \frac{(r-\theta)(1-T)}{(\theta + p - r)(r+p)}\right)$$

But if we want to retain our old definitions—W for wealth *not including government bonds,* and T being the old tax rate—then the law of motion for wealth is:

$$\frac{dW_t}{dt} = (r - \theta - p)\left(W_t + G - \frac{(r-\theta)(1-T-rG)}{(\theta + p - r)(r+p)}\right)$$

The transfer leads to a fall in the long-run steady-state value of other wealth.

f. What does this model tell us about the debate over Ricardian equivalence? (No more than one page, please!)

It shows just one of the many ways to break Ricardian equivalence: in this case, that mortality—the fact that one may not be around to pay the taxes needed to finance the transfer that enriches one now—means that the timing of taxes and transfers does have an effect on consumption.

3. Exchange Rate Crises: Suppose that the demand for money balances in a small open economy is given by:

$$m_t - e_t = -\eta \times \frac{de_t}{dt}$$

where m is the (log) money stock, e is the (log) exchange rate, and η is a parameter. Suppose that the government does not have power to print banknotes: it must work through the central bank, which conducts open-market operations—sales of bonds for banknotes, or purchases of bonds for banknotes—and foreign-exchange interventions—sales of domestic bonds for foreign bonds, or purchases of domestic bonds for foreign bonds. For simplicity's sake, suppose that the interest the central bank earns on its bond holdings is just sufficient to cover the costs of running the central bank.

This problem comes from one of the problem sets for Ken Rogoff's undergraduate international macroeconomics course at Harvard.

a. Explain why, with this institutional setup for the central bank, before any speculative attack the money stock is equal to:

$$M_t = B_t + E^* F_t$$

where B is the central bank's net holdings of domestic bonds, E* is the (fixed) level of the exchange rate, and F is the central bank's net holdings of foreign exchange reserves.

The only way that money can be created or reduced in this economy is through open market operations—thus dM/dt must equal dB_{om}/dt, where "om" means the change through open market operations. The only way that the central bank can lose or gain foreign exchange reserves is by buying or selling them for bonds. Hence dF/dt must equal -(1/e*)dB_{fe}/dt, where "fe" means the change through foreign exchange interventions. If these two conditions hold always, then the central bank balance sheet is as above...

b. Suppose that the central bank keeps the log exchange rate fixed at e*. What path must the money stock follow in order to maintain this exchange rate target?

With a fixed exchange rate the money stock must be constant and equal to E*, the level of the fixed exchange rate target.

c. Suppose that government fiscal policy is such as to require that the central bank increase its holdings of domestic bonds at a proportional rate θ as long as it maintains its fixed exchange rate. What is the time path of the central bank's holdings of foreign exchange reserves as long as the fixed exchange rate target is maintained?

We know that the central bank must keep the money stock constant if it is to keep the exchange rate fixed. It must also add to its stock of domestic bonds at a proportional rate θ, which implies exponential growth as long as the fixed exchange rate system lasts. These two imply that:

$$\frac{dF_t}{dt} = -\frac{1}{E^*}\frac{dB_t}{dt} = -\frac{\theta B_t}{E^*}$$

$$B_t = B_0 e^{\theta t}$$

$$F_t = F_0 + \frac{B_0 - B_0 e^{\theta t}}{E^*}$$

As long as the central bank starts out with positive holdings of domestic bonds, foreign exchange reserves will decline. And eventually they will decline far enough to trigger a speculative attack.

When a speculative attack takes place the government responds by selling its foreign-currency reserves for domestic bonds, and then turning around and selling the excess domestic bonds for cash in order to keep it on its domestic debt accumulation path. Thus a speculative attack does not cause a discontinuous change in B, but it does cause a discontinuous change in M.

Suppose that after a speculative attack government fiscal policy also changes—so that after the shock of the attack central bank holdings of domestic bonds grow not at a proportional rate θ but at some alternative rate φ. And suppose that everyone in this economy has *perfect foresight*.

d. Suppose φ=θ and both are greater than zero. Will this economy suffer a speculative attack on its exchange rate? If so, when?

Yes, it will. The speculative attack when it takes place will reduce the money stock by the current amount of foreign exchange reserves. Thus if you define the shadow exchange rate e_t^s to be:

$$e_t^s = b_t + \eta\phi = b_0 + \theta t + \eta\phi$$

Then the speculative attack will occur the moment that this shadow exchange rate equals e*. It cannot happen any earlier—the people trading domestic for foreign bonds will then suffer a capital loss on their foreign bond holdings, because if it happens earlier the exchange rate will suddenly and discontinuously appreciate. That can't happen in a perfect-foresight model. It cannot happen any later—if it happens any later everyone holding domestic bonds suffers a capital loss, and that cannot happen in a perfect foresight model.

e. Suppose θ is greater than zero, but that $\varphi=0$. Will this economy suffer a speculative attack on its exchange rate? Why or why not? What happens after the central bank's foreign exchange reserves are exhausted?

It won't really be an attack: because the post-"attack" inflation rate is zero, the post-"attack" log exchange rate will be constant at e*. Thus money demand will not be affected. Foreign exchange reserves will smoothly decline to zero. And at that point fiscal policy will change...

f. Suppose $\theta=0$, but that φ is greater than zero. Can this economy suffer a speculative attack? If so, under what circumstances can it suffer a speculative attack? Will it suffer one?

Yes, it can suffer a speculative attack. If:

$$F_0 = M_0 - E^* e^{-\eta\phi}$$

then a speculative attack *can* (but does not have to) occur at any moment. The change in fiscal policy validates the attack whatever moment it occurs, even though having the attack never occur is a perfectly fine perfect-foresight equilibrium too.

If:

$$F_0 < M_0 - E^* e^{-\eta\phi}$$

then the speculative attack should have occurred already, and will occur at time zero (with capital losses for those holding domestic assets).

If:

$$F_0 > M_0 - E^* e^{-\eta\phi}$$

g. Suppose that you are in charge of writing a memo for a non-economist who has just become head of your country's central bank and wants to know what he or she should do to guard against a speculative attack on the currency. How does your analysis above inform what you write? What do you say? (One page only, please!)

This problem shows the lessons of first and second generation models of currency crisis. A central bank trying to maintain a fixed exchange rate when the rest of the government is running an unsustainable and permanent inflationary deficit cannot do so indefinitely. That is the lesson of part d. The lesson of part e is that the end of one's foreign exchange reserves need not imply devaluation and depreciation thereafter: if policy shapes up, you can maintain the e* level of the log exchange rate even without any foreign exchange reserves at all.

The lesson of part f is exactly the reverse: even if your current policies are sustainable, a belief that after a speculative attack policies will be inflationary *may* trigger a speculative attack—and the change in policy that *ex post* validates the attack. It does not have to trigger one. But it may. To maintain a fixed exchange rate, you may need to convince markets not only that current policies are sustainable, but that the government's preferences will not shift and become more inflationary after a successful speculative attack. You may not: lots of reserves—a value of F_0 high enough—can substitute for a lack of credibility.