# Suggested Problem Set 2 Solutions

# Economics 202B – Second Half

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**9.14 Money versus interest rate targeting.** (Poole, 1970.)
Setup:

$$\begin{aligned} y &= c - ai + \varepsilon_{IS}, \text{ IS curve} \\ m - p &= hy - ki + \varepsilon_{LM}, \text{ LM curve} \\ E(\varepsilon_{IS}) &= E(\varepsilon_{LM}) = 0, Var(\varepsilon_{IS}) = \sigma_{IS}^2, Var(\varepsilon_{LM}) = \sigma_{LM}^2, p = \bar{p} \end{aligned}$$

**(a)** If $i = \bar{i}$ then we simply take the variance of the IS curve and obtain:

$$Var(y) = Var\left(c - a\bar{i} + \varepsilon_{IS}\right) = \sigma_{IS}^2.$$

**(b)** If $m = \bar{m}$ then we can solve for $y$ as a function of the two disturbance terms and constants as follows:

$$\begin{aligned} i &= \frac{hy - \bar{m} + \bar{p} + \varepsilon_{LM}}{k}, \text{ from the IS and substitute into LM:} \\ y &= c - \frac{a}{k}(hy - \bar{m} + \bar{p} + \varepsilon_{LM}) + \varepsilon_{IS}, \text{ which solving for } y \text{ yields:} \\ y &= \frac{k}{ah+k}(c + \varepsilon_{IS}) - \frac{a}{ah+k}(\bar{p} + \varepsilon_{LM} - \bar{m}). \end{aligned}$$

Therefore, $Var(y) = \left(\frac{k}{ah+k}\right)^2 \sigma_{IS}^2 + \left(\frac{a}{ah+k}\right)^2 \sigma_{LM}^2$.

**(c)** $\sigma_{IS}^2 = 0$, therefore

$$Var^{(a)}(y) = 0 < Var^{(b)}(y) = \left(\frac{a}{ah+k}\right)^2 \sigma_{LM}^2,$$

so *interest rate* targeting is best.

**(d)** $\sigma_{LM}^2 = 0$, therefore

$$Var^{(a)}(y) = \sigma_{IS}^2 > Var^{(b)}(y) = \left(\frac{k}{ah+k}\right)^2 \sigma_{IS}^2,$$

so *money* targeting is best.

**(e)** The intuition for part (c) where there are only LM shocks is as follows. If the policymaker targets the nominal money stock, the LM shocks cause the LM curve to shift around and equilibrium output in the economy is determined by the intersection of that shifting LM curve with the stable IS curve. If the policymaker targets the nominal interest rate, it ensures that $i$ remains constant in the face of any LM shock. Since $i$ is not allowed to change, planned expenditure does not change and thus the level of output that equates planned and actual expenditure does not change in the face of an LM shock.

The intuition for part (d) where there are only IS shocks is as follows. If the policymaker targets the nominal interest rate, equilibrium output changes by the full extent of the shift in the IS curve caused by a shock to the IS curve. Now, consider the case where the policymaker targets the nominal money stock. A positive IS shock shifts the IS curve to the right. With $m$ fixed, as $y$ rises to equate planned and actual expenditure, $i$ rises as well in order for the money market to remain in equilibrium. This rise in $i$ reduces planned expenditure and thus mitigates some of the positive shock. Therefore, $y$ does not end up rising as much. The same logic holds for a negative IS shock. In this case, $y$ does not fall as much as if the policymaker had kept $i$ constant.

**(f)** If there are only IS shocks, then it is possible to keep $y$ constant at some target level $\hat{y}$. Therefore, $Var(y) = 0$ which is less than either interest rate or money targeting. By re-arranging the LM curve with $y$ set to $\hat{y}$, the nominal money supply must be such that: $m = p + h\hat{y} - ki$. The policymaker knows the fixed $p$, has picked $\hat{y}$ herself and can observe $i$. Therefore, when $i$ changes (and we know this change is coming from only IS shocks) the policymaker must change $m$ accordingly. For example, as $i$ rises $m$ must be reduced.

**9.15 Uncertainty and policy.** (Brainard, 1967.)
Setup:

$$y = x + (k + \varepsilon_k)z + u, \text{ output, where } z \text{ is policy instrument}$$
$$E(\varepsilon_k) = E(u) = 0, Var(\varepsilon_k) = \sigma_k^2, Var(u) = \sigma_u^2, u \text{ and } \varepsilon_k \text{ are independent}$$

**(a)** Find $E\left[(y - y^*)^2\right]$:

$$\begin{aligned} E\left[(y-y^*)^2\right] &= Var(y-y^*) + [E(y-y^*)^2] \\ &= Var(x + kz + \varepsilon_k z + u - y^*) + [E(x + kz + \varepsilon_k z + u - y^*)]^2 \\ &= (z^2\sigma_k^2 + \sigma_u^2) + (x + kz - y^*)^2, \end{aligned}$$

where the last step uses the fact that $\varepsilon_k$ and $u$ have mean zero.

**(b)** The policy maker wishes to minimize the solution in part (a) w.r.t. $z$:

$$\frac{\partial E\left[(y-y^*)^2\right]}{\partial z} = 2z\left(k^2+\sigma_k^2\right) + 2k\left(x-y^*\right) = 0, \text{ so}$$
$$z^* = \frac{k\left(y^*-x\right)}{k^2+\sigma_k^2}.$$

**(c)** To determine how the policymaker should respond to shocks, a change in $x$, take the derivative of $z^*$ to arrive at

$$\frac{\partial z^*}{\partial x} = -\frac{k}{\sigma_k^2+k^2} < 0.$$

Therefore, an increase in $x$ should be offset by a higher $z$. Next, note that $\sigma_u^2$ does not affect the policymaker's response to a change in $x$. Hence, the optimal degree of "fine-tuning" does not depend on the amount of uncertainty about the state of the economy.

**(d)** By inspection of the solution in part (c) one can see that an increase in $\sigma_k^2$ decreases the scope for "fine tuning" given uncertainty about the state of the economy. Formally:

$$\frac{\partial[\partial z^*/\partial x]}{\partial \sigma_k^2} = \frac{k}{\left(\sigma_k^2+k^2\right)^2} > 0.$$

Therefore, the more uncertain the effects of policy are, the less scope there is for policy.

**9.17** (Cagan, 1956.)
Setup:

$$m(t) = Ce^{-b\pi^e(t)} \quad (1)$$
$$\dot{\pi}^e(t) = \beta[\pi(t) - \pi^e(t)], \;\; 0 < \beta < 1/b \quad (2)$$
$$S(t) = g_M(t)m(t), \;\; G = g_M(t)m(t) \quad (3)$$
$$\pi(t) = g_m(t) - \frac{\dot{m}(t)}{m(t)} \quad (4)$$

**(a)** The following steps, using the equations from the setup, will yield an expression for $\dot{\pi}^e(t)$ as a function of $\pi^e(t)$. First, take the time derivative of (1) to get

$$\dot{m}(t) = -b\dot{\pi}^e(t)Ce^{-b\pi^e(t)}. \quad (5)$$

Next, divide both sides of (5) by $m(t)$ which yields

$$\frac{\dot{m}(t)}{m(t)} = -b\dot{\pi}^e(t). \quad (6)$$

Now, substitute (6) and the equation for $g_M(t)$ from (3) to arrive at:

$$\pi(t) = \frac{G}{m(t)} + b\dot{\pi}^e(t). \tag{7}$$

Finally, substitute (7) into (2) and re-arrange terms to arrive at:

$$\dot{\pi}^e(t) = \frac{\beta}{1-\beta b}\left[\frac{G - \pi^e(t)m(t)}{m(t)}\right]. \tag{8}$$

**(b)** If we assume that $G > S^*$ then $G > \pi^e m$ for all $\pi^e$. Therefore, since $\beta b < 1$ (8) is always positive, so expected inflation will grow without bound regardless of where it begins. To get an idea of how the phase diagram looks, substitute $m(t) = Ce^{-b\pi^e(t)}$ into (8) and take first and second derivative with respect to $\pi^e(t)$ to yield

$$\frac{d\dot{\pi}^e(t)}{d\pi^e(t)} = \frac{\beta b G e^{b\pi^e(t)}}{(1-\beta b)C} - \frac{\beta}{1-\beta b}, \tag{9}$$

$$\frac{d^2\dot{\pi}^e(t)}{d\pi^e(t)^2} = \frac{\beta b^2 G e^{b\pi^e(t)}}{(1-\beta b)C} > 0. \tag{10}$$

By setting (9) to 0 we note that $\dot{\pi}^e(t)$ reaches its minimum at $\pi^e(t) = [ln(C/bG)]/b$. See Figure 1 below.

**(c)** Now, if $G < S^*$ there are two possible growth rates for money consistent in raising the amount $G$ in seignorage (refer to Figure 9.8 in the text). Call these two rates $g_1$ and $g_2$, where $g_1 < g_2$. At steady-state, inflation equals expected inflation, so since by assumption $\pi^e(t)m(t) = G$, $\pi^e(t) = g_1$ and $\pi^e(t) = g_2$. From (8) $\dot{\pi}^e(t) = 0$ at $\pi^e(t) = g_1$ and $\pi^e(t) = g_2$. One can see from Figure 2. (see below) that low-inflation steady-state $\pi^e(t) = \pi(t) = g_1$ is stable, while the high-inflation steady-state $\pi^e(t) = \pi(t) = g_2$ is unstable.

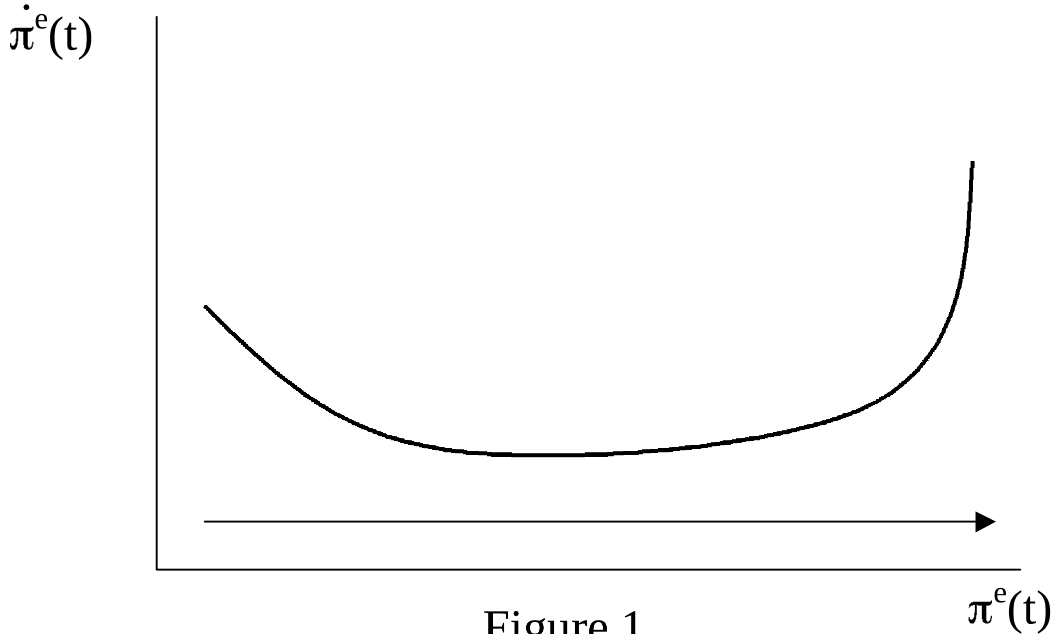


Figure 1.

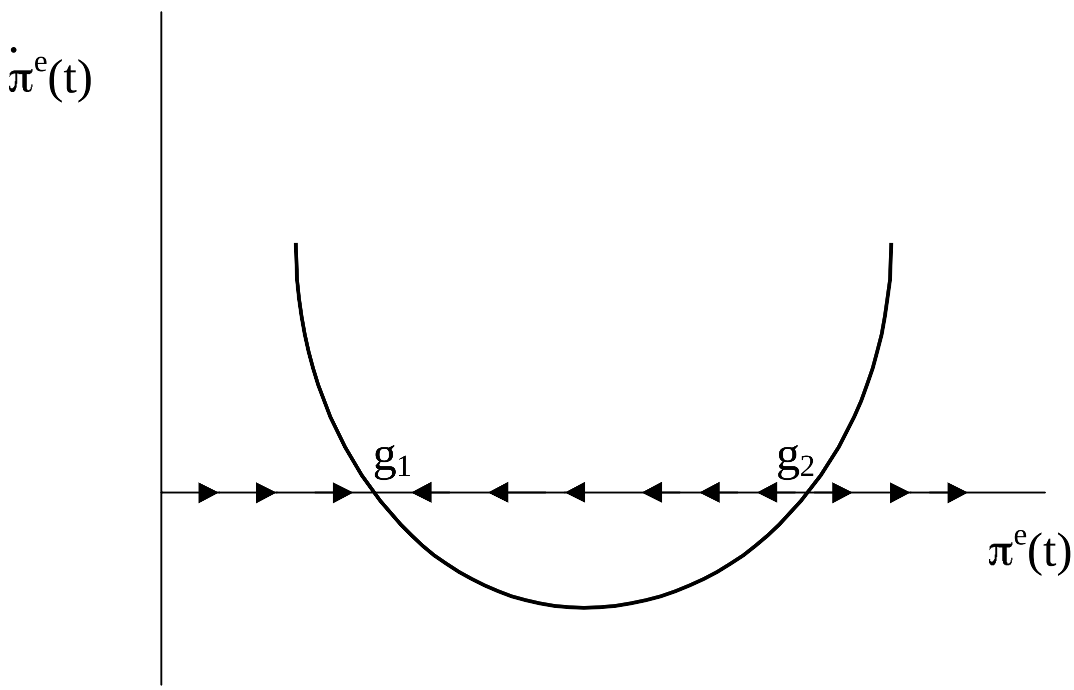


Figure 2.