Suggested Problem Set 4 Solutions
Economics 202B – Second Half

Prepared by Julian di Giovanni*

11.11 Alesina-Drazen model. Comparative statics.

(a) As T falls, $V'(A) = \frac{1}{B-A}[B-(W-T)-2A]$ will be smaller. So if initially $V'(A) \geq 0$, and the change in T is small enough, $V'(A)$ will stay positive and so by the FOC of the worker's maximization problem:

$$X^* = \frac{B-(W-T)}{2}. \tag{1}$$

So, workers' offer, X^*, will fall with the cost, T, since $\frac{\partial X^*}{\partial T} = .5 > 0$. This will affect the probability of reform, $P(X^*)$:

$$\frac{\partial P(X^*)}{\partial T} = \frac{-1}{2(B-A)} < 0.$$

Therefore, the probability of reform will rise. If, instead, the change in T is not small enough, $V'(A)$ will become negative and so workers' offer will be set equal to A and the reform will always be implemented.

If initially $V'(A) < 0$, the initial workers' offer, X^*, will be A and the reform will always be carried out. Since $V'(A)$ will be smaller as T falls, both workers' offer, X^*, and the probability of reform, $P(X^*)$, will stay constant.

(b) As B rises, the marginal utility of worker's utility at $X = A$, $V'(A) = \frac{1}{B-A}[B-(W-T)-2A]$, will also rise since

$$\frac{\partial V'(A)}{\partial B} = \frac{(W-T)+A}{(B-A)^2} > 0.$$

Therefore, if initially $V'(A) \geq 0$, the FOC of the worker's maximization problem will still be given by (1). So, workers' offer, X^*, is increasing in the profit upper bound, B, since $\frac{\partial X^*}{\partial B} = .5 > 0$. This will affect the probability of reform, $P(X^*)$:

$$\begin{aligned}
\frac{\partial P(X^*)}{\partial B} &= \frac{(1-.5)(B-A)-1(B-X^*)}{(B-A)^2} \\
&= \frac{-.5A-.5B+X^*}{(B-A)^2} \\
&= \frac{-.5A-.5B+.5B-.5(W-T)}{(B-A)^2} \\
&= -\frac{A+(W-T)}{2(B-A)^2} < 0.
\end{aligned}$$

*I thank Andrea De Michelis for allowing me to use and edit the solution to 11.11.

So, the probability of reform will fall.

If initially $V'(A) < 0$, the initial workers' offer, X^*, is set to A and the reform always occur. Since $V'(A)$ rises with B, if the increase in B is large enough, $V'(A)$ will become positive. Hence, workers' offer will be higher than A and the probability of reform will be lower than 1. On the contrary, if the rise in B is not large enough, the workers' offer will stay constant at A and the reform will always be implemented.

(c) If A and B rise by the same amount, $V'(A) = \frac{1}{B-A}[B - (W - T) - 2A]$ will be smaller, so if initially $V'(A) > 0$, and the change in A and B is small enough, $V'(A)$ will stay positive and so the FOC of the worker's maximization problem will still be given by (1). So, workers' offer, X^*, will increase, and will by less than B. The probability of a reform will also increase since $P(X^*) = \frac{B-X^*}{B-A}$, and the numerator will be greater while the denominator will be unchanged. If, instead, the changes in A and B are not small enough, $V'(A)$ will become negative. In this case, the workers' offer will be set equal to A and the reform will always occur.

If, initially, $V'(A) \leq 0$, the workers' offer, X^*, will remain A and the probability of reform, $P(X^*)$, will remain 1.

11.12 Alesina-Drazen model. Conditionality and reform.

Here, if reform is implemented both workers and capitalists will be given $F > 0$ by an international agency. We will first solve for X^* and $P(X^*)$ and then consider the change in social welfare from this aid policy.

The probability of acceptance is now given by:

$$P(\Pi + F > X) = P(X) = \begin{cases} 1, & \text{if } X \leq A + F; \\ \frac{B-(X-F)}{B-A}, & \text{if } A + F < X < B + F; \\ 0, & \text{if } X \geq B + F. \end{cases}$$

Hence, the expected utility of workers is

$$V(X) = \begin{cases} (W - T) + X + F & \text{if } X \leq A + F; \\ \frac{B-(X-F)}{B-A}\left[(W - T) + X + F\right], & \text{if } A + F < X < B + F; \\ 0, & \text{if } X \geq B + F. \end{cases}$$

Note that for $X \leq A + F$, workers' expected utility is increasing in F and for $X > B + F$, it is negative. Therefore, since workers set X to maximize their expected utility, $X^* \in [A + F, B + F]$. The partial derivative of $V(X)$ with respect to X is

$$V'(X) = \frac{B - (W - T) - 2X}{B - A}.$$

Now, if $V''(X) < 0$ at $X = A + F$, workers will offer $X^* = A + F$. However, if $V''(X) > 0$ at $X = A + F$, workers will offer X^* such that $V'(X^*) = 0$. Therefore,

$$X^* = \begin{cases} A + F, & \text{if } V'(A+F) < 0; \\ \frac{B-(W-T)}{2}, & \text{if } V'(A+F) \geq 0. \end{cases}$$

Next, note that the probability of the capitalists accepting the offer X^* is

$$P(X^*) = \begin{cases} 1, & \text{if } V'(A+F) < 0; \\ \frac{B-(W-T)+2F}{2(B-A)}, & \text{if } V'(A+F) \geq 0. \end{cases}$$

(a) Therefore, to solve the first part of the problem, simply take the derivative of $P(X^*)$ (when reform is not already for certain) to arrive at

$$\frac{\partial P(X^*)}{\partial F} = \frac{1}{B-A} > 0,$$

which implies foreign aid always increases the probability of reform.

(b) To show that social welfare function is always increasing with reform we must consider two cases: (1) where reform is certain, and (2) when reform occurs with a probability less than one.

1. Define the expected social welfare function as $S(X^*) =$ Workers' Expected Payoff + Capitalists' Expected Payoff . Then if $X^* = A + F$ capitalists accept the workers' offer with probability one and expected social welfare is

$$\begin{aligned} S(X^*) &= (W - T) + F + E(\Pi) + F \\ &= W - T + \frac{A+B}{2} + 2F, \end{aligned}$$

which implies that

$$\frac{\partial S(X^*)}{\partial F} = 2 > 0,$$

so increasing foreign aid always increases expected social welfare if reform is certain to begin with.

2. Next, consider when reform is not for certain. Then, Workers' Expected Payoff is

$$\begin{aligned} E(Pay^W) &= P(X^*)[(W-T) + X^* + F] \\ &= \frac{B + (W-T) + 2F}{2(B-A)}[(W-T) + X^* + F], \end{aligned} \tag{1}$$

and Capitalists' Expected Payoff is

$$
\begin{aligned}
E(Pay^C) &= \underbrace{\int_A^{X^*-F} 0\frac{1}{B-A}d\Pi}_{\text{No reform}} + \underbrace{\int_{X^*-F}^{B} (\Pi - X^* + F)\frac{1}{B-A}d\Pi}_{\text{Reform}} \\
&= \frac{1}{B-A}\left[\Pi^2 - (X^* - F)\Pi\right]_{X^*-F}^{B} \\
&= \frac{B^2 - (X^* - F)B}{B-A}. \qquad (2)
\end{aligned}
$$

Summing the expected payoffs of workers and capitalists, (1) and (2) respectively, we arrive at expected social welfare:

$$
S(X^*) = \frac{B + (W-T) + 2F}{2(B-A)}[(W-T) + X^* + F] + \frac{B^2 - (X^* - F)B}{B-A},
$$

and taking the derivative of this expression with respect to F we arrive at

$$
\frac{\partial S(X^*)}{\partial F} = \frac{3[B + (W-T)] + 2(X^* + F)}{2(B-A)} > 0,
$$

which proves that an increase in foreign aid always increases expected social welfare even when reform is not certain.

11.14 Status-quo bias. (Fernandez and Rodrik, 1991.)

Setup: There are two policies, A and B, which are voted on. A fraction f of the population know whether they are strictly better of under each policy; a fraction α is better of under A, and a fraction $1-\alpha$ under B. The remaining $1-f$ of the population know that a fraction β of them are better off under A, and a fraction $1-\beta$ under B. The payoffs are symmetric, i.e., plus 1 if better off under A and minus 1 if worse off under A. A re-vote is possible where all individuals now know their own type.

(a) For the case where fraction $1-f$ of the population knows only that fraction β of them are better off under A, then the fraction of the population who prefers A, F^A is

$$
F^A = \begin{cases} \alpha f, & \text{if } \beta < \frac{1}{2}; \\ \alpha f + (1-f), & \text{if } \beta > \frac{1}{2}. \end{cases}
$$

The intuition for this results is the following. First, note that we know in the f fraction of the population that α percent prefer A and know it, therefore

αf will always vote for A. Now, if the $1-f$ fraction of the population believe that on average this part of the population prefers B, i.e., $\beta < \frac{1}{2}$ then each individual's expected payoff in this fraction of the population is greater if they vote for B, so no one votes for A. Now, if $\beta > \frac{1}{2}$ the converse is true, so everyone (i.e., $1-f$) votes for A.[1]

(b) Now, if all individuals know under which policy they are better off under, the fraction of the population that prefers A can be expressed simply as

$$F^A = \alpha f + (1-f)\beta,$$

since now each individual in the fraction $1-f$ of the population know under which policy she is better off under, and the fraction β prefers A.

(c) In answering this question I shall only examine the cases where Policy A is initially in effect (symmetric arguments can be made for Policy B). There are two possible situations: (1) the majority votes for A in the first place, in which case no more voting occurs, and (2) the majority first votes for B, the fraction $1-f$ of the population then learn its type, and the majority re-votes for A.[2] We must also consider whether $\beta < \frac{1}{2}$ or $\beta > \frac{1}{2}$.

1. $\beta < \frac{1}{2}$: Policy A is retained in a first vote if $\alpha f > \frac{1}{2}$.[3]
   $\beta > \frac{1}{2}$: Policy A is retained in a first vote if $\alpha f + (1-f) > \frac{1}{2}$.

2. $\beta < \frac{1}{2}$: Policy B is first voted for if $\alpha f < \frac{1}{2}$. Policy A then wins in a re-vote if $\alpha f + (1-f)\beta > \frac{1}{2}$. Combining these two conditions yields the condition $(1-f)\beta > 0$.
   $\beta > \frac{1}{2}$: Policy B is first voted for if $\alpha f + (1-f) < \frac{1}{2}$. Policy A then wins in a re-vote if $\alpha f + (1-f)\beta > \frac{1}{2}$. Combining these two conditions yields the condition $(1-f)(1-\beta) < 0$, which never holds. Therefore, policy A will never be retained in this case.

---

[1] I am ignoring the case when $\beta = \frac{1}{2}$, in which case the fraction $1-f$ of the population is indifferent between A and B.

[2] The second case never actually occurs given the assumption made concerning costs of voting – policy A will be retained in the first vote. However, we must still consider the conditions under which this event would occur – either way policy A is retained.

[3] I am ignoring the possibilities of ties, Florida not-withstanding.

11.15 The common-pool problem in government spending. (Weingast, Shepsle, and Johnsen, 1981.)
Setup:

$$U_i = E + V(G_i) - C(T), \text{ utility for district } i$$
$$V'(\bullet) > 0,\ V''(\bullet) < 0,\ C'(\bullet) > 0,\ C''(\bullet) > 0$$
$$\sum_{i=1}^{M} G_i = MT, \text{ government's budget constraint, } M > 1,\ T_i = T\ \forall\ i$$

(a) To find the first-order condition for the value G_j chosen by the representative district j, taking the spending of other districts as given, simply solve the following maximization problem:

$$\max_{G_j} E + V(G_j) - C(T)$$
$$\text{s.t. } T = \frac{1}{M}\left(G_j + \sum_{i \neq j}^{M} G_i\right).$$

The first order condition for this problem is

$$V'(G_j) = \frac{C'(T)}{M}. \tag{1}$$

(b) To find the first-order condition for the Nash equilibrium value of G simply solve the following maximization problem:

$$\max_{G} E + V(G) - C(T)$$
$$\text{s.t. } T = \frac{1}{M}\sum_{i=1}^{M} G.$$

The first order condition for this problem is

$$V'(G) = C'(T). \tag{2}$$

(c) The Nash equilibrium is Pareto efficient. One can see this by simply noting that the first-order condition (2) implies that in equilibrium each district equates the marginal gain from local public spending to the marginal loss from taxes. This is not the case in equation (1) where the marginal gain is in fact less then the marginal cost. This results from the externality of district spending in this economy. Each district doesn't take into account how its spending affects the total tax bill and hence the tax charged to each district. This results in over-spending at the district level. The Nash equilibrium internalizes this externality.