Solutions to First Midterm
Econ 101B

September 25 2003

Question 1:
a. Steady-state capital-output ratio: $\kappa^* = \frac{s_p + sg}{n+g+\delta} = \frac{0.24}{0.08} = 3$
Balanced-growth path of output per worker:

$$y_{BG} = (\kappa^*)^{\frac{\alpha}{1-\alpha}} E_0 e^{0.03t} = 60,000e^{0.03t}$$

b. It's growing at rate $g = 3\%$. In fact $\left.\frac{d\ln(y)}{dt}\right|_{BG} = 0.03$.
c. $y_{BG}^{new} = \left(\frac{0.2}{0.08}\right)^1 \times 20,000e^{0.03t} = 50,000e^{0.03t}$.
d. The rate of convergence is $(1-\alpha)(n+g+\delta) = 4\%$. Hence, a reasonable approximation is:

$$\kappa_1 \approx \kappa_0 - 0.04(\kappa_0 - \kappa_{new}^*) = 3 - 0.04(3 - 2.5) = 2.98$$

In alternative, if you want to be precise use the expression for the law of motion:

$$\kappa_1 = (\kappa_0 - \kappa_{new}^*)e^{-0.04} + \kappa_{new}^* = 2.9804$$

e. Just replace in the given expression:

$$y_1 = \frac{Y_1}{L_1} = (\kappa_1)^{\frac{\alpha}{1-\alpha}} E_1 = 2.98 \times 20,000e^{0.03} \approx \$61,415$$

Question 2:
a. As in the usual Solow model: $\kappa^* = \frac{s}{n+g+\delta} = \frac{0.2}{0.05} = 4$.
b. $\kappa_{new}^* = \frac{s}{p_k(n+g+\delta)} = \frac{0.2}{0.025} = 8$.
To see this note:

$$\begin{aligned}
\frac{d\ln(\kappa)}{dt} &= \frac{d\ln(K)}{dt} - \frac{d\ln(Y)}{dt} = \frac{d\ln(K)}{dt} - \alpha\frac{d\ln(K)}{dt} - (1-\alpha)(n+g) = \\
&= (1-\alpha)\left[\frac{d\ln(K)}{dt} - (n+g)\right] = (1-\alpha)\left[\frac{sY}{p_k K} - (n+\delta+g)\right] = \\
&= (1-\alpha)\left[\frac{s}{p_k\kappa} - (n+\delta+g)\right]
\end{aligned}$$

So, in steady-state, $\frac{d\ln(\kappa^*)}{dt} = 0 \Leftrightarrow \kappa^*_{new} = \frac{s}{p_k(n+g+\delta)}$.

c. Had there been no change in p_k, the output per worker after one year would have been

$$y_1^{old} = 4^1 \times 20,000 \times e^{0.01} = \$80,804$$

As in Question 1. part (e), the new output per worker after one year will depend on the rate of convergence $(1-\alpha)(n+g+\delta) = 2.5\%$:

$$y_1^{new} = \kappa_1 E_0 e^{0.01} \approx [4 + 0.025(8-4)] \times 20,000 \times e^{0.01} = \$82,824$$

Hence, the growth rate is:

$$\frac{y_1^{new} - y_0}{y_0} = \frac{\$82,824 - \$80,000}{\$80,000} = 3.53\%$$

d. Because the economy's output per worker is above its original balanced-growth path they'll conclude that a combination of any of the following happened: a rise in s, a rise in g, a fall in n, or a fall in δ. This is because all of these parameter changes either increase the level (through κ^*) or the slope of the balanced-growth path of output per worker.

e. In principle, the parameters n, s, and δ are directly observable, or can be quantified from the national accounts and the census data. Over the long run, because a change in p_k only affects the level of balanced-growth path of y, it would be distinguishable from a change in g that affects both the level and the slope of y_{BG}.

Question 3:

a. $\kappa^* = \frac{0.2}{0.05} = 4$; $y_{BG} = (\kappa^*)^{\frac{\alpha}{1-\alpha}} E_0 = \sqrt[3]{4} \times 20,000 = \$31,800$.

Balanced-growth path of consumption per worker:

$$c_{BG} = (1-s)y_{BG} = 0.8 \times 31,800 = \$25,440.$$

b. $\kappa^* = \frac{0.25}{0.05} = 5$; $y_{BG} = \sqrt[3]{5} \times 20,000 = \$34,200$ and

$$c_{BG} = 0.75 \times 34,200 = \$25,650$$

c. $\kappa^* = \frac{0.3}{0.05} = 6$; $y_{BG} = \sqrt[3]{6} \times 20,000 = \$36,400$ and

$$c_{BG} = 0.7 \times 36,400 = \$25,480$$

d. I would choose $s = 30\%$ to maximize output per worker.

e. I would now choose $s = 25\%$, because it maximizes the level of consumption per worker. If we accept that people's objective is to reach as high a level of utility as possible, and that utility depends on their consumption of goods and services, we can approximate "social welfare" by the level of consumption per worker.

Question 4:
Topics of answer

- differences in population growth (depending on stage of demographic transition)
- differences in savings and investment rates, and adverse terms of trade for poor countries that have to buy capital goods for a high relative price
- acceleration of growth of efficiency of labor through time (as in Kremer 1993), and differences in the access and openness to imported technology and know-how
- the argument about the "development of underdevelopment"
- differences in levels of education achievement, and technical competence of the population
- differences in economic policies and broad institutional settings (political regimes, law systems), with impact on economic incentives
- maybe different geographic and climatic conditions.