Solutions to Problem Set 1

Econ 101B - Fall 2003

1. 1. Yes, counted as a negative investment (disinvestment).

   2. Yes, because the fees represent the value added of the service provided by the real estate agent.

   3. No, social security checks are a form of transferring back to households money collected through taxes. They redistribute income, but do not add to national income.

   4. Yes, because all government purchases of goods and services are included in GDP. A new aircraft carrier should show up as a piece of public investment.

   5. Yes, since rents are the counterpart to the purchase of "housing services" provided by landlords.

   6. Yes. GDP is a measure of production, not of consumption or purchases, so that it also includes the goods and services sold to foreigners (but not the goods and services bought from abroad).

2. Real GDP = Nominal GDP/Price level.

3. Real interest rate (r) = Nominal interest rate (i) - inflation (π)

4. The calculations for the two cases are different because it is usual to approximate $(1+r)(1+\pi) = 1+i \Leftrightarrow 1+r = \frac{1+i}{1+\pi}$ with $r \approx i - \pi$. Other then that, the calculations are identical if you only notice that $1+\pi_t = P_t/P_{t-1}$ and let $g^N(g^R)$ be the growth rate of nominal (real) GDP, and $Y^N(Y^R)$ nominal (real) GDP, so that we can rewrite:

   $$Y_t^R = \frac{Y_t^N}{P_t} = \frac{(1+g^N)Y_{t-1}^N}{(1+\pi)P_{t-1}} \Rightarrow \frac{Y_t^N/P_t}{Y_{t-1}^N/P_{t-1}} = \frac{1+g^N}{1+\pi} \Leftrightarrow \frac{Y_t^R}{Y_{t-1}^R} = 1+g^R = \frac{1+g^N}{1+\pi}$$

   which we could also approximate with $g^R \approx g^N - \pi$.

5. Annual rate of inflation $\pi_{79} = \frac{55.22}{50.88} - 1 = 8.53\% \Rightarrow r_{79} = 10\% - 8.53\% = 1.47\%$.

6. For 1998, $r_{98} = 4.8\% - 2.6\% = 2.2\%$, higher then in 1979. So, as this example shows, borrowers and lenders should care about real interest rates, because nominal rates (depending on the current inflation) may be very misleading of the actual real purchasing power that borrowers pay lenders in the form of interest payments. In extreme cases, real interest rates are negative, i.e., interest payments are not even sufficient to return the lender the same purchasing power he lent to the borrower initially.

7. (a) \$600.
   (b) Change in business inventories.
   (c) \$150.
   (d) Consumption: \$750, Change in business inventories: -\$600.

8. A model in Economics serves to reduce the complexity and variety of reality to a manageable number of behavioral relationships and equilibrium conditions that can be worked out analytically. If properly specified, the model should contribute to grasp the general principles underlying the real-world economy.

9. There is a direct correspondence between an algebraic identity (an equation) and a graph that represents the relation between the variables in that equation. The axis of the graph should represent the variables included in the equation.

10. In the first place there are the several imperfections of the GDP in measuring the final goods and services produced in the economy, namely, because of the practical incapacity in getting a reliable measure of depreciation, the inconsistent treatment of government purchases, and several omissions (household production, depletion, pollution, and other "bads"). A second flaw is that GDP per worker is only an average measure that ignores the distribution of productivity levels. Nevertheless, GDP per worker is still the most reliable summary statistic of the material welfare allowed, on average, to the workers of the economy, hence its widespread use.

11. Time to double: $1.03^t = 2 \Leftrightarrow t = \frac{\ln 2}{\ln(1.03)} \approx 23.45$ years (**remember**: the quantity is growing at a proportional not an exponential rate).

    Time to quadruple: $t = \frac{\ln 4}{\ln(1.03)} \approx 46.9$ years.

    Time to grow 1024-fold: $t = \frac{\ln 1024}{\ln(1.03)} \approx 234.5$ years.

12. Long-run steady-state: $\frac{dx(t)}{dt} = 0 \Leftrightarrow x^* = -.32/.08 = -4$.

    Time to halve distance to steady-state: we know (see section notes of 08/26) that the distance of $x(t)$ from the steady-state decays at a constant rate of .08. So, the time to halve this distance solves $e^{-.08t} = 1/2 \Leftrightarrow t \approx 8.66$ years.

    Time to close 3/4 of initial distance: $e^{-.08t} = 1/4 \Leftrightarrow t \approx 17.33$ years.

Time to close 7/8 of distance: $e^{-.08t} = 1/8 \Leftrightarrow t \approx 25.99$ years.

Time to close 15/16 of distance: $e^{-.08t} = 1/16 \Leftrightarrow t \approx 34.66$ years.

13.

$$\begin{aligned} \frac{dx}{dt} &= -.08x - .32 \Leftrightarrow \int e^{.08t} \left(\frac{dx}{dt} + .08x \right) dt = -.32 \int e^{.08t} dt \Leftrightarrow \\ &\Leftrightarrow e^{.08t} x = -4e^{.08t} + c \Leftrightarrow x = -4 + e^{-.08t} c \end{aligned}$$

where c is a constant of integration.

If $x(0) = 6 \Rightarrow 6 = -4 + c \Leftrightarrow c = 10$, and general solution: $x = -4 + 10e^{-.08t}$.

Analogously, if $x(0) = 8 \Rightarrow c = 12$, and $x = -4 + 12e^{-.08t}$.

14. The growth rate of y is $g(y) = \frac{y_t}{y_{t-1}} - 1 = \left(\frac{x_t}{x_{t-1}} \right)^a - 1 = 1.06^a - 1$

So, if $a = .25 \Rightarrow g(y) = 1.06^{.25} - 1 \approx 1.46\%$. Or we could approximate (as if y grew exponentially) by $g(y) = .25 g(x) = 1.5\%$.

If $a = .5 \Rightarrow g(y) = \sqrt{1.06} - 1 \approx 2.95\%$.

If $a = 1 \Rightarrow g(y) = 6\%$.

If $a = 2 \Rightarrow g(y) = 1.06^2 - 1 = 12.36\%$.

15. Taking logs and derivatives: $\ln z = b (\ln x - \ln y) \Leftrightarrow g(z) = b [g(x) - g(y)]$.

If $g(x) = .06, g(y) = 0, b = 1/3 \Rightarrow g(z) = 2\%$.

If $g(y) = .02 \Rightarrow g(z) = 1\frac{1}{3}\%$.

If $g(y) = .06 \Rightarrow g(z) = 0$.

If $g(y) = .12 \Rightarrow g(z) = -2\%$.