Solutions to Problem Set 2

Econ 101B Spring 2003

1. (a) $\frac{Y}{N} = \left(\frac{64}{100}\right)^{\frac{1}{3}} \approx 0.862$.

   (b) $\frac{Y}{N} = \left(\frac{49}{196}\right)^{\frac{1}{3}} 3^{2/3} \approx 1.31$.

   (c) $Y = K^a (EL)^{1-a}$. So, if K and L both double, output will be: $(2K)^a (2LE)^{1-a} = 2Y$, which means that the production function exhibits constant returns to scale.

   (d) With $\frac{K}{L} = 2 \Rightarrow \frac{Y}{L} = 2^a$, whereas if $\frac{K}{L} = 4 \Rightarrow \frac{Y}{L} = 4^a$, and if $\frac{K}{L} = 6 \Rightarrow \frac{Y}{L} = 6^a$.

   Notice that the growth rate of $\frac{Y}{L}$ is just the growth rate of $\frac{K}{L}$ times the parameter a. For instance, when $\frac{K}{L}$ increases 100% from 2 to 4, $\frac{Y}{L}$ increases by $100 \times a\%$. To see this just apply the rule of the growth rate of a power:

$$g(y) = \frac{\frac{dy}{dt}}{y} = \frac{\frac{dy}{dk}\frac{dk}{dt}}{y} = \frac{ak^{a-1}\frac{dk}{dt}}{k^a} = a\frac{\frac{dk}{dt}}{k} = ag(k)$$

   in which $y = \frac{Y}{L}$, and $k = \frac{K}{L}$.

2. The balanced growth path of output per worker is given by $\left(\frac{Y}{L}\right)_{BG} = \left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}} E$. Hence, if δ falls the balanced growth path of $\frac{Y}{L}$ shifts upward.

3. (a) Barring other capital market adjustments (coming from private savings or the capital account), this implies a fall in s to 0.15. So, the new steady-state capital-output ratio is:

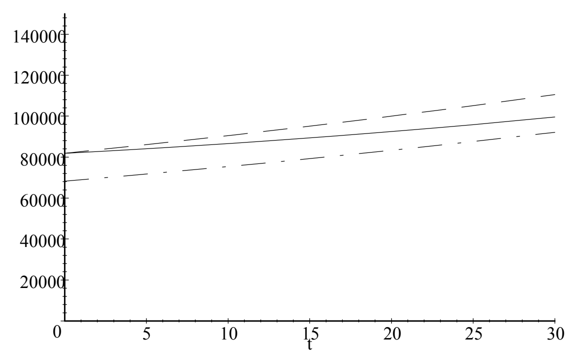
$$\kappa^{**} = \frac{0.15}{0.02 + 0.01 + 0.03} = 2.5$$

   (b) Again, along the balanced growth path: $y = (\kappa^*)^{\frac{\alpha}{1-\alpha}} E$. Before the shift in the budget *deficit* this was $y_{BG}^{old} = 3^{\frac{\alpha}{1-\alpha}} E$. After the shift it is $y_{BG}^{old} = 2.5^{\frac{\alpha}{1-\alpha}} E$, i.e., steady state output per worker will converge toward a lower time path (although keeping on growing at rate $g = 0.01$).

(c) First notice that the initial forecast of \$100,000 (call it $\widehat{y}_{20}^{old}$) should have been gotten from $\widehat{y}_{20}^{old} = y_0 e^{0.01\times 20} = (\kappa^*)^{\frac{\alpha}{1-\alpha}} E_0 e^{0.01\times 20}$.
After 20 years the economy will probably still be converging to its new steady-state. Using the approximation mentioned on page 109 of the textbook, we can say that approximately $20 \times (1-\alpha)(n+g+\delta)\% = 1.2(1-\alpha)$ of the initial distance from the steady-state $(\kappa^* - \kappa^{**})$ will have been closed. Meanwhile E keeps on growing at rate g, so that we can make a new forecast of:

$$\begin{aligned} \widehat{y}_{20}^{new} &= \left[\kappa^{**} + (\kappa^* - \kappa^{**})\, e^{-1.2(1-\alpha)}\right]^{\frac{\alpha}{1-\alpha}} E_{20} = \\ &= \left[2.5 + (3-2.5)\, e^{-1.2(1-\alpha)}\right]^{\frac{\alpha}{1-\alpha}} E_0 e^{0.2} \end{aligned}$$

If you think the algebra is getting too messy assume $\alpha = 1/2$, in which case we can solve for all variables of the model, and hence, for $\widehat{y}_{20}^{new}$. In particular, we can back up E_0 from the expression for the initial estimate: $\$100,000 = 3E_0 e^{0.2} \Leftrightarrow E_0 = \$27291$. And then, $\widehat{y}_{20}^{new} \approx \$92480$.
$27291\left[2.5 + (3-2.5)\, e^{-0.6}\right] e^{0.2} = 92480.0$
The graph shows the convergence (solid line) from the old (large dash) to the new (small dash) balanced growth paths:



4. This mandatory replacement investment has a similar implication to the model as a one-time rise in depreciation. It reduces the steady-state capital-output ratio, and hence, the path of output per worker. Still, as the polluting capital stock is replaced, the "depreciation" will decrease until it converges back to the original value (when all the capital stock has been replaced). So, over the long run, output per worker will return to its original path. In this case, the economy sustains a loss in output level during the period of replacement of capital and after, while it returns to the old balanced growth path. To interpret this as a "wealth" loss for the economy is, however, not so clear-cut namely because, with the new capital stock, the economy will be spared the negative externalities from pollution.

5. The immediate effect is to reduce output per worker, because, starting from the steady-state $\frac{Y}{L} = \left[\left(\frac{K}{Y}\right)_{BG}\right]^{\frac{\alpha}{1-\alpha}} E$, and neither K nor E adjust discontinuously. Total output, on the other hand, increases since $Y = K^{\alpha}(EL)^{1-\alpha}$. As Y rises above the initial value, the capital-output ratio falls bellow its steady-state value, but the economy will converge back to the original $\left(\frac{K}{Y}\right)_{BG}$ and balanced growth path of $\frac{Y}{L}$.

   The opposition from the AFL-CIO can be understood from the fact that, other things equal, immigration lowers output per worker, and this may have a depressing impact on wages. The marked acceleration of productivity growth in the second half of the 1990s probably justifies the easing of the position of the AFL-CIO regarding immigration.

6. Over the long run, the growth will be the same, namely, g. The growth of total output will be smaller, though, because along the steady-state, $g(Y) = n + g$. During the transition there will be a levels effect because a smaller n will mean the convergence to a higher steady-state value of $\frac{K}{L}$ and to a higher path of $\frac{Y}{L}$.

7. (a) The old steady-state capital-output ratio was $\frac{0.2}{0.05} = 4$, whereas the new is $\frac{0.2}{0.06} = 3.33$, a 16.66% reduction.

   (b) $\widehat{y}^{old}_{2043} = y_{2003} \times \$15,000 \times e^{0.01\times 40} = 4^{.5} \times \$15,000 \times e^{0.01\times 40} = \$44,755$. Assuming that by 2043 the economy would already have converged to its new balanced growth path (the time to cover half of the distance being approximately 17 years) we can forecast:

   $\widehat{y}^{new}_{2043} = 3.33^{.5} \times \$15,000 \times e^{0.02\times 40} = \$60,949$. Notice how the depressing effect of the rise of g on the steady-state capital-output ratio is more than compensated by the higher growth rate of labor efficiency.

8. (a) $\left(\frac{K}{Y}\right)_{SS} = \frac{0.2}{0.05} = 4$.

   (b) $\left(\frac{Y}{L}\right)_{2003} = \left(\frac{K}{Y}\right)_{SS}^{\frac{\alpha}{1-\alpha}} \times E = 4 \times \$20,000 = \$80,000$.

   (c) Below.

   (d) $(1-a)(n+g+d) = 0.025$, i.e., each year approximately 2.5% of the gap is covered. Then, $y_{2004} = \$70,000 + 0.025(\$80,000 - \$70,000) = \$70,250$.

   (e) $y_{2013} - \$80,000 = (\$70,000 - \$80,000)e^{-0.25} \Leftrightarrow y_{2013} = \$72,212$.