Solutions to Problem Set 3

Econ 101b Spring 2003

1. Same as in PS #2, question 6 (*repetitio est mater studiorum :-)*.

2. One would answer that the acceleration in the pace of innovation and consequence increase in the efficiency of labor, especially after the middle 18th century, allowed for a steady increase in standards of living *and* population (you can think of Kremer's argument in this context). One could also mention the industrial revolution as marking not only an acceleration, but also a change in the process of invention and its conversion into economic applications.

3. (a) Original $\kappa^* = \frac{0.2}{0.05} = 4$, new $\kappa^{**} = \frac{0.2}{0.065} \approx 3.08$ (a decrease of 23%).
   (b) $\widehat{y}^{old}_{2040} = y_{2004}E_{2040} = 4^{0.5} \times \$16,500e^{0.01\times 36} \approx \$47,300$
   $\widehat{y}^{new}_{2040} = (3.08)^{0.5} \times \$16,500e^{0.025\times 36} \approx \$71,224$.

4. Part a. doesn't change. As for part b:
   If $\alpha = 1/2$
   $\widehat{y}^{old}_{2040} = 4 \times \$16,500e^{0.01\times 36} \approx \$94,600$
   $\widehat{y}^{new}_{2040} = 3.08 \times \$16,500e^{0.025\times 36} \approx \$124,997$
   If $\alpha = 2/3$
   $\widehat{y}^{old}_{2040} = 4^2 \times \$16,500e^{0.01\times 36} \approx \$378,399$
   $\widehat{y}^{new}_{2040} = 3.08^2 \times \$16,500e^{0.025\times 36} \approx \$384,990$.

5. In here I think one should distinguish between *proximate* and *underlying* sources of divergence in productivity and per capita income levels. The first are relatively easy to identify since there's a strong correlation between levels of output per worker and the determinants of its balanced growth path, namely investment rate (positive correlation), population growth (negative), and variables influencing the level and growth of the efficiency of labor, e.g., access/openness to new technologies and the present overall education level.

   Harder is to identify what may explain why some countries have higher investment rates, lower population growth, and better conditions for improving the efficiency of labor. Economists have tried several alternative explanations: impact of the degree of openness, geo-climatic factors, institutions of governance, or political structures.

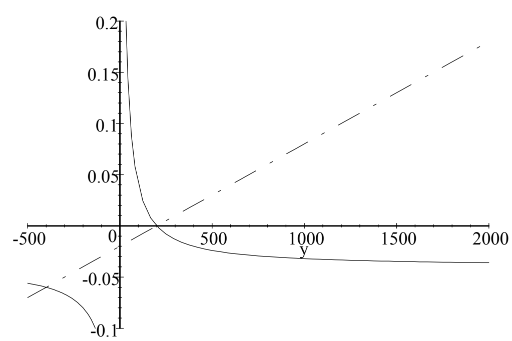
6. (a) With $\alpha = 1 \Rightarrow \frac{Y}{L} = \frac{K}{Y}$, which means output per worker is directly determined by the capital-output ratio. In alternative, you may notice that this is equivalent to the production function $Y = K^{0.5}L^{0.5}$. If by "distant future" we mean "on the balanced growth path", this implies that output per worker will be constant and equal to the steady-state capital-output ratio: $\frac{Y}{L} = \frac{s}{n+\delta} = \frac{0.2}{0.04} = 5$. Notice that g doesn't show up in the formula for κ^* (this can be easily seen from Prof DeLong's derivation at the start of the 09/09 lecture).

   (b) $\widehat{y}^{new} = \frac{0.15}{0.04} = 3.75$.

   (c) None.

   (d) With $\alpha = 1$ the growth of labor efficiency has no impact on output, which rules out the constant long run growth rate of y. Since there are still decreasing returns in the accumulation of capital the economy converges to a steady-state instead of a balanced growth path, where is y constant. This is a way of seeing why Solow had to introduce the variable "efficiency of labor" growing at a constant rate. If he didn't so he wouldn't get long run (per worker) growth, but only transitory movements towards the steady-state whenever the economy was pushed out of it.

7. (a) The expressions plotted are:
   $\frac{Y}{L} = y = 100\left(\frac{0.08}{n+0.04}\right) \Leftrightarrow n = \frac{8}{y} - 0.04$ (solid line)
   $n = 0.0001\,(y - 200)$(dashed).



   (b) The crossing points solve $\frac{Y}{L} = 100\left(\frac{0.08}{0.0001\left(\frac{Y}{L}-200\right)+0.04}\right) \Leftrightarrow \frac{Y}{L} = -400 \vee \frac{Y}{L} = 200$. Ignoring the negative root, the curves cross at $\frac{Y}{L} = 200, n = 0$. These are also the equilibrium values of output per worker and population growth.

   (c) By "short run" interpret "before population adjusts", so that $\frac{dy}{ds} = \frac{100}{0.04} = 2500$ (also assuming we start from the steady state).

Over the "long run" population also adjusts. Start from the implicit definition of equilibrium: $y = 100\left(\frac{s}{0.0001(y-200)+0.04}\right)$ and differentiate with respect to s:

$$\begin{aligned} \frac{dy}{ds} &= \frac{100(0.0001y + 0.02) - 100s \times 0.0001 dy/ds}{(0.0001y + 0.02)^2} \Leftrightarrow \\ &\Leftrightarrow \frac{dy}{ds} = \frac{2 + 0.01y}{(0.0001y + 0.02)^2 + 0.01s} \end{aligned}$$

For instance, starting from the steady-state with $s = 0.08$, we get $\frac{dy}{ds} = 1666.66$, not surprisingly smaller than the short run effect. Just think that an initial rise in the savings rate makes $\frac{Y}{L}$ rise above the Malthusian level of \$200, and will be partially counterbalanced by an increase in population.