Solutions to Problem Set 7

Econ 101B Fall 2003

1. $S = \sum_{t=1}^{T}(u_t - u^*) \underset{\text{Phillips curve}}{\underset{\downarrow}{=}} \frac{1}{\beta}\sum_{t=1}^{T}(\pi_{t-1} - \pi_t) = [(\pi_0 - \pi_1) + (\pi_1 - \pi_2) + ...$

   $+(\pi_{T-2} - \pi_{T-1}) + (\pi_{T-1} - \pi_T)] \Rightarrow S = \frac{\pi_0 - \pi_T}{\beta}$

   S is measured in annualized percentage unemployment, i.e., you can choose to reduce inflation by sustaining an unemployment rate of $S + u^*$ in just one year, or by spreading the cost through the T years, with an unemployment rate of $S/T + u^*$ in each year.

2. Sacrifice ratio $= \frac{S}{\pi_0 - \pi_T} = \frac{\pi_0 - \pi_T}{\beta(\pi_0 - \pi_T)} = \frac{1}{\beta}$.

   I would be best advised in trying to estimate the sensitivity of inflation to the unemployment gap (the parameter β in the Phillips curve). If you follow Prof. DeLong's argument in pages 332-33, β can be decomposed into the coefficient of Okun's law and the slope of the short-run aggregate supply curve. Overall, I should also try to gain an idea of the shape of the Phillips curve (namely whether it is convex or not - recall the section on the Phillips curve).

3. Start again from:

$$S = \sum_{t=1}^{T}(u_t - u^*) = \sum_{t=1}^{T}\frac{\pi_t^e - \pi_t}{\beta}$$

   Now substitute recursively for the series of expected inflation: π_1^e, π_2^e, π_3^e,...π_T^e by assuming that the economy starts from equilibrium in year 0, i.e. $\pi_0^e = \pi_0$:

$$\begin{aligned}
\pi_1^e &= \pi_0 \\
\pi_2^e &= (1-\lambda)\pi_1^e + \lambda\pi_1 = (1-\lambda)\pi_0 + \lambda\pi_1 \\
\pi_3^e &= (1-\lambda)\pi_2^e + \lambda\pi_2 = (1-\lambda)^2\pi_0 + \lambda(1-\lambda)\pi_0 + \lambda\pi_2 \\
&\;\;... \\
\pi_T^e &= (1-\lambda)\pi_{T-1}^e + \lambda\pi_{T-1} = (1-\lambda)^{T-1}\pi_0 + \lambda(1-\lambda)^{T-2}\pi_1 + \\
&\quad +\lambda(1-\lambda)^{T-3}\pi_2 + ... + \lambda\pi_{T-1}
\end{aligned}$$

Replace this series into the expression for S:

$$\begin{aligned} S &= \frac{\pi_1^e - \pi_1}{\beta} + \frac{\pi_2^e - \pi_2}{\beta} + \frac{\pi_3^e - \pi_3}{\beta} + ... + \frac{\pi_T^e - \pi_T}{\beta} = \\ &= \frac{\pi_0 - \pi_1}{\beta} + \frac{(1-\lambda)\,\pi_0 + \lambda\pi_1 - \pi_2}{\beta} + \frac{(1-\lambda)^2\,\pi_0 + \lambda\,(1-\lambda)\,\pi_0 + \lambda\pi_2 - \pi_3}{\beta} ... \end{aligned}$$

Factor the common terms:

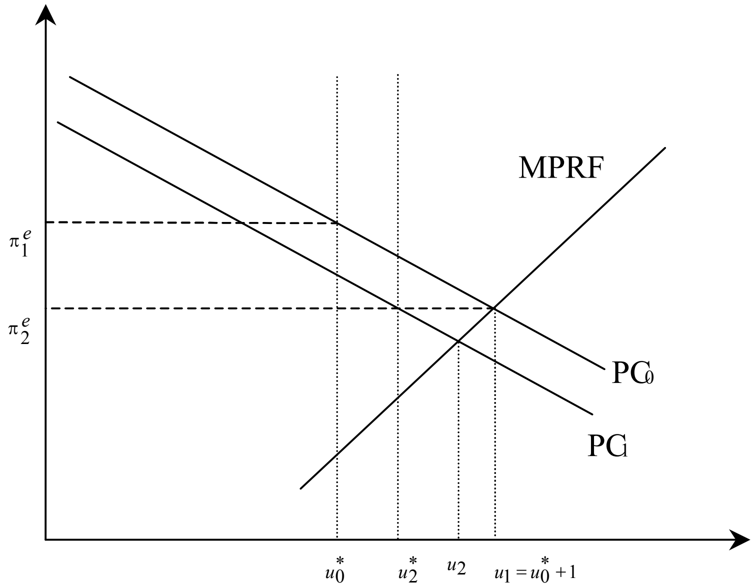
$$\begin{aligned} \beta S &= \left[1 + (1-\lambda) + (1-\lambda)^2 + ... + (1-\lambda)^{T-1}\right]\pi_0 - \left[1 - \lambda - \lambda(1-\lambda) - ... - \lambda\,(1-\lambda)^{T-2}\right]\pi_1 - \\ &\quad - \left[1 - \lambda - \lambda(1-\lambda) - ... - \lambda\,(1-\lambda)^{T-3}\right]\pi_2 - ... - (1-\lambda)\,\pi_{T-1} - \pi_T \end{aligned}$$

Now use the properties of the sum of geometric progressions to condense a bit the expression:

$$S = \frac{1}{\beta}\left[\frac{1-(1-\lambda)^T}{\lambda}\pi_0 - (1-\lambda)^{T-1}\pi_1 - (1-\lambda)^{T-2}\pi_2 - ... - (1-\lambda)\,\pi_{T-1} - \pi_T\right]$$

So, contrary to question 1, where the cumulative excess of unemployment S only depended on the initial and final levels of inflation, it is now a weighted average of all the intermediate levels of inflation. The same remark carries through for question 2: I would now also have to estimate the coefficient of expectations adjustment (λ).

4. First of all notice that this is a model of *hysteresis*, because the natural rate of unemployment depends on past unemployment rate. This implies that the central bank, by fighting inflation through a recession raises the natural rate of unemployment. After year 1 the central bank leaves the economy to converge back to the "natural rate". However, this is not the original natural rate, but some other natural rate of unemployment unto which the economy will converge by the mechanism of adaptive expectations. In the figure below I represent the first two iterations of the process, when the inflation rate falls from π_0 to π_1 and then to π_2, while the natural rate of unemployment rises from $u_0^* = u_1^*$ to u_2^*.



Algebraically, we can calculate the time-path of inflation and unemployment by solving recursively. Before that we need the expression of the MPRF. We know that $u_1 = u_0^* + 1$, and replacing in the Phillips curve, we get $\pi_1 = \pi_0 - \beta$. The MPRF crosses this point. Hence, we can back up its expression from this point and assume it has a slope ϕ, as usual. That is:

$$u_t = u_0^* + 1 + \phi\left(\pi_t - \pi_0 + \beta\right)$$

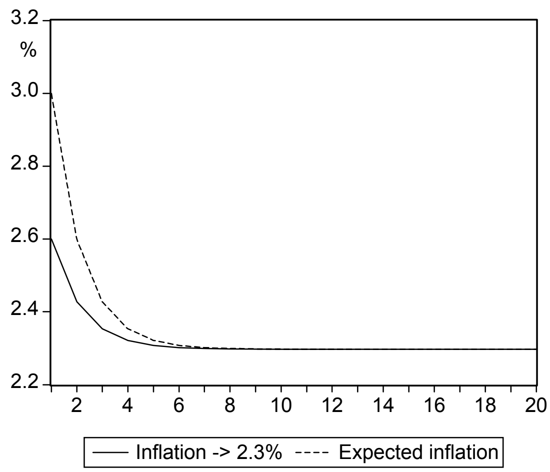
Now, solve, for instance, for the time path of inflation π_t by replacing in the Phillips curve u_t^* with its definition and u_t from the MPRF:

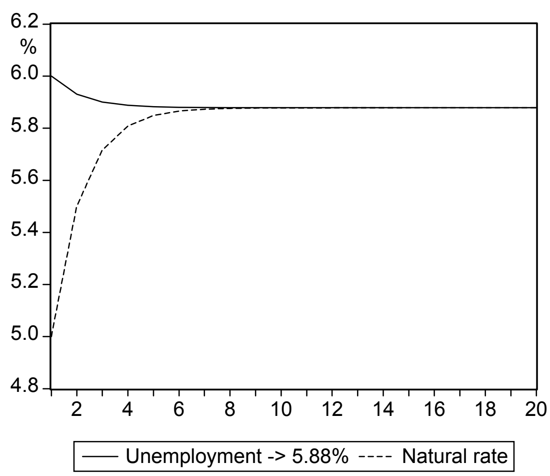
$$\begin{aligned} \pi_t &= \pi_{t-1} + \beta\left[(1-\theta)\, u_{t-1}^* + \theta u_{t-1} - \phi\left(\pi_t - \pi_0 + \beta\right) - u_0^* - 1\right] \Leftrightarrow (...) \Leftrightarrow \\ &\Leftrightarrow \pi_t = \frac{1}{1+\beta\phi}\pi_{t-1} + \frac{\beta}{1+\beta\phi}\left[(1-\theta)\, u_{t-1}^* + \theta u_{t-1}\right] + \frac{\beta}{1+\beta\phi}\left[\phi\left(\pi_0 - \beta\right) - u_0^* - 1\right] \end{aligned}$$

You will notice that this expression only depends on *lagged* terms and parameters, so we can solve recursively for π_t, starting from π_0. Then, just replace the values of π_t in the MPRF to get the time path of u_t. For the first three years the expressions are the following:

Year	π	u	u^*
1	$\pi_0 - \beta$	$u_0^* + 1$	u_0^*
2	$\pi_0 - \beta + \frac{\beta(\theta-1)}{1+\beta\phi}$	$u_0^* + \theta - \frac{\theta-1}{1+\beta\phi}$	$u_0^* + \theta$
3	$\pi_0 - \beta + \frac{\beta(\theta-1)}{(1+\beta\phi)^2}(\beta\phi - \theta + 2)$	$u_0^* + \theta - \frac{\theta(\theta-1)}{1+\beta\phi} + \frac{(\theta-1)^2}{(1+\beta\phi)^2}$	$u_0^* + \theta - \frac{\theta(\theta-1)}{1+\beta\phi}$

This recursive system will converge to a stable solution with a new natural rate of unemployment and expectations of inflation coinciding with actual inflation. As an example, the following two figures represent the time paths of these variables for a special choice of parameters and starting values.[1] Note the convergence towards a stable solution (2.3% inflation and 5.88% unemployment).





5. That would depend on the structure of the economy, namely on the original natural rate of unemployment and the hysteresis parameter θ. If θ is very small it may be acceptable to fight inflation for a small cost in terms of structural unemployment. Actually, this cost might even be reversed later, during an upswing, when the central bank could let the economy fall below the then natural rate for some time and use the hysteresis effect to

[1] $u_0^* = 5\%$, $\pi_0 = 3\%$, $\beta = \phi = 0.4$, and $\theta = 0.5$.

reduce the natural rate. Nevertheless, my intuition would be that it is always very costly to tamper with the *structural* rate of unemployment and that the long term costs of this policy of disinflation may not be justified by the current benefits.

6. If the central bank targets the natural unemployment rate, inflation will permanently rise to $\pi_0 + s$. This is because in year 2 and after expectations coincide with the actual rate of inflation the central bank has to accept to sustain the natural rate of unemployment. If instead the central bank targets the initial inflation level (and assuming we start from the natural rate) unemployment in the first year rises s/β above the natural rate. However, in the next period to keep inflation at π_0 the central bank has to adopt an expansionary policy that moves the economy back to the natural rate of unemployment, where it stays absent further shocks.

7. Beyond having to get an estimate of the relevant parameters and variables of the model (in particular of s and β) I would have to make an assessment of the trade-off between having a permanently higher inflation or having a temporarily higher unemployment. This probably would depend on the political costs of the two alternatives.

8. Now, with unemployment targeting, there is only a spike in the time path of the inflation rate, in the first year, as after the shock the Phillips curve returns to its original position. The answer with inflation targeting is the same (obviously).

9. Just refer to the difference in the response of inflation to unemployment targeting between questions 6 and 8. An "inflation hawk" is more likely to enjoy static expectations and be able to fight unemployment (keep it at its natural rate) at a lower cost in terms of inflation.

10. Judgemental: no absolute right answer.