1. Consider an economy well modeled by the standard growth model, with a labor force growth rate of 2% per year, a depreciation rate of 4% per year, a rate of growth of the efficiency of labor of 2% per year, and a savings rate of 16% of GDP.

   a. Suppose that the diminishing-returns-to-capital parameter $\alpha$ is $1/3$. What is the proportional increase in the steady-state level of output per worker generated by an increase in the savings rate from 16% to 17%?

   b. Suppose that the diminishing-returns-to-capital parameter $\alpha$ is $1/2$. What is the proportional increase in the steady-state level of output per worker generated by an increase in the savings rate from 16% to 17%?

   c. Suppose that the diminishing-returns-to-capital parameter $\alpha$ is $2/3$. What is the proportional increase in the steady-state level of output per worker generated by an increase in the savings rate from 16% to 17%?

   d. Suppose that the diminishing-returns-to-investment capital $\alpha$ is $3/4$. What is the proportional increase in the steady-state level of output per worker generated by an increase in the savings rate from 16% to 17%?

2. Consider the Solow growth model, in which output per worker along the steady-state growth path is given by:

   $$\frac{Y_t}{L_t} = (\kappa^*)^\lambda \times E_t$$

   where the steady-state capital-output ratio $\kappa^*$ is given by:

   $$\kappa^* = \frac{s}{n + g + \delta}$$

   where the growth multiplier $\lambda$ is a simple function of the diminishing-returns-to-scale parameter $\alpha$ in the production function:

   $$\lambda = \frac{\alpha}{1 - \alpha}$$
and where $s$ is the economy’s savings-investment rate, $n$ is the labor force growth rate, $\delta$ is the depreciation rate, and $g$ is the proportional rate of growth of the efficiency of labor $E_t$.

d. Suppose that we have two economies A and B, identical save that in economy A the efficiency of labor is twice that of economy B. What is the ratio of their steady-state output per worker levels?

e. Suppose that at some point in the past economies A and B had had the same levels of the efficiency of labor, but that labor efficiency in A had then begun growing at 2% points per year faster than in B. About how many years would it take before economy A had twice the labor efficiency of economy B?

f. Now change the situation slightly. Suppose that in addition to the difference in efficiency of labor levels, economy A has a savings-investment rate 1.5 times as large as economy B because investment goods tend to be cheaper in rich countries. If the diminishing-returns-to-scale parameter $\alpha = 2/3$, what is the ratio of the two economies’ steady-state output per worker levels?

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• Explain why—using words only—the impact of the difference in efficiency-of-labor levels (and the consequences of that difference) so much bigger in the fourth part than it was in the third part.

3. Suppose that population growth depends on the level of output per worker, so that:

\[ n = (.0001) \times [(Y/L) - \$200] \]

the population growth rate $n$ is zero if output per worker equals $\$200$, and that each $\$100$ increase in output per worker raises the population growth rate by 1% per year.

Suppose also that the economy is in its *Malthusian* regime, so that the rate of increase of the efficiency of labor $E$ is zero. Thus output per worker is given by:
\[ \frac{Y_t}{L_t} = \left( \frac{s}{n + \delta} \right)^{\frac{\alpha}{1 - \alpha}} E_0 \]

with the diminishing-returns-to-investment parameter \( \alpha = .5 \), with the depreciation rate \( \delta = .04 \), and with the efficiency of labor \( E_0 = $100 \).

4. Suppose somebody who hasn't taken any economics courses were to ask you why humanity escaped from the Malthusian trap--of very low standards of living and slow population growth rates that nevertheless put pressure on available natural resources and kept output per worker from rising--in which humanity found itself between the year 8000 B.C.E. and 1800. What answer would you give? (One paragraph only, please!)