Growth Accounting, Natural Resources, and Pre Civil War America

U.C. Berkeley Economics 113 Spring 2005 Handout

Introduction

We would like to have a way to understand and think about just how and why different economies grow at different rates over the long run. In the context of Economics 113—American Economic History—we have a definite puzzle: it was Britain that was ahead in technology and was where technology was moving ahead the fastest in the first half of the nineteenth century, and yet it was America that appears to have had the fastest per-person economic growth. According to eh.net, British growth in real GDP per capita averaged 0.50% per year in the first half of the nineteenth century; American real GDP per capita growth averaged 0.86% per year from 1790-1850.[1]

So let's get a formula for the growth of GDP in terms of various potential causes in order to begin analyzing this question. And let's first of all shift our attention from GDP per capita to GDP per worker: the ratio of workers to population is not something that we believe changes, and GDP per worker is more straightforward to analyze.

Math

Let's begin with the simple, standard, Cobb-Douglas production function, which economists use because it is a flexible form that can adequately model a great many situations:

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[1] See Louis Johnston and Samuel H. Williamson, "The Annual Real and Nominal GDP for the United States, 1789 - Present." Economic History Services, March 2004, URL : http://www.eh.net/hmit/gdp/; Lawrence H. Officer, "The Annual Real and Nominal GDP for the United Kingdom, 1086 - 2000." Economic History Services, June 2003, URL : http://www.eh.net/hmit/ukgdp/.

$$\frac{Y}{L} = A\left(\frac{K}{L}\right)^{\alpha}\left(\frac{N}{L}\right)^{\beta}$$

Where:

A:	an index of the level of technology and organization.
K/L:	the capital-labor ratio.
N/L:	natural resources per worker.
Y/L:	output per worker.
α:	a parameter between 0 and 1: the importance of capital in production.
b:	a parameter between 0 and 1: the importance of resources in production.

Let's use lowercase letters to refer to per-worker quantities—to something divided by the labor force L:

$$y = Ak^{\alpha}n^{\beta}$$

And let's take (natural) logs: logarithms of everything to the base e = 2.71828… (why do we do this? Because natural log changes are growth rates, and proceeding this way produces numbers that are exceedingly easy to interpret; why does expressing everything as a power of 2.71828… produce numbers that are exceedingly easy to interpret? That may be a deep question; it may be a silly question; in either case, we don't have a terribly helpful and clear explanation):

$$\ln(y) = \ln(A) + \alpha\left[\ln(k)\right] + \beta\left[\ln(n)\right]$$

Now let's take derivatives with respect to time—and remember that the time derivative of the natural log of a variable is one over the variable times the rate of change of the variable:

$$\frac{\partial}{\partial t}\left[\ln(x)\right] = \frac{1}{x}\frac{\partial x}{\partial t}$$

And get:

$$\frac{1}{y}\frac{\partial y}{\partial t} = \frac{1}{A}\frac{\partial A}{\partial t} + \alpha\left[\frac{1}{k}\frac{\partial k}{\partial t}\right] + \beta\left[\frac{1}{n}\frac{\partial n}{\partial t}\right]$$

Where:

$\frac{1}{y}\frac{\partial y}{\partial t}$ *is the proportional growth rate of GDP per worker.*

$\frac{1}{A}\frac{\partial A}{\partial t}$ *is the proportional growth rate of technology and organization.*

$\frac{1}{k}\frac{\partial k}{\partial t}$ *is the proportional growth rate of capital per worker.*

$\frac{1}{n}\frac{\partial n}{\partial t}$ *is the proportional growth rate of natural resources per worker.*

α *is the importance of capital in production.*

β *is the importance of natural resources in production.*

For reasons you don't need to worry about (unless you are also taking Econ 100b or 101b or PEIS 107 right now), in the long run GDP per worker and capital per worker ought to grow at about the same proportional rate:

$$\frac{1}{y}\frac{\partial y}{\partial t} = \frac{1}{k}\frac{\partial k}{\partial t}$$

So we can rewrite our growth rate equation as:

$$(1-\alpha)\frac{1}{y}\frac{\partial y}{\partial t} = \frac{1}{A}\frac{\partial A}{\partial t} + \beta\left[\frac{1}{n}\frac{\partial n}{\partial t}\right]$$

And then divide through by 1-α to get:

$$\frac{1}{y}\frac{\partial y}{\partial t} = \left\{\frac{\frac{1}{A}\frac{\partial A}{\partial t} + \beta\left[\frac{1}{n}\frac{\partial n}{\partial t}\right]}{1-\alpha}\right\}$$

Application

We now have a way of talking about the importance of three different factors in contributing to the growth of output per worker. Growth in output per worker will be faster…

- the larger is α, the importance of capital in production (for a larger α means a smaller 1-α, and thus a larger quotient). We have no strong reason to believe that α was bigger in the United States. The belief that profits were a larger share of income in England tends to suggest that it was smaller.
- the faster is the rate of improvement in technology and organization. But here Britain—the heart of the industrial revolution—has the edge.
- the larger is the importance of natural resources in production (b) and the faster is the rate at which accessible resources grow relative to population.

In the first half of the nineteenth century British GDP per capita grew at 0.50% per year. Let's assign a value of 0.25 to β, the importance of resources in production; a value of 0.4 to α, the importance of capital in production; and assign –0.75% per year as the trend growth rate of British resources per worker (it's a small island with well-surveyed resources and a growing population). Then our equation:

$$\frac{1}{y}\frac{\partial y}{\partial t} = \left\{ \frac{\frac{1}{A}\frac{\partial A}{\partial t} + \beta\left[\frac{1}{n}\frac{\partial n}{\partial t}\right]}{1-\alpha} \right\}$$

becomes:

$$0.5\% = \left\{ \frac{\frac{1}{A}\frac{\partial A}{\partial t} + 0.25\left[-0.75\%\right]}{1-0.4} \right\}$$

which implies a proporational growth rate of technology and organization A in Britain of 0.49% per year.

If we are willing to assign a growth rate one-tenth of a percentage point lower to A in the United States—0.39% per year—then the similar equation for America becomes:

$$0.86\% = \left\{ \frac{0.39\% + 0.25\left[\frac{1}{n}\frac{\partial n}{\partial t}\right]}{1 - 0.4} \right\}$$

which requires a value of 0.52% per year for the proportional rate of growth of natural resources per capita in America. During the 1790-1850 period when the population of the United States increases six-fold—from 4 to 25 million—it looks as though available natural resources increased eightfold, generating a one-third increase in available resources per capita.

The westward expansion—the Erie Canal, the steamboats, expulsion of Indians from the near midwest and the inland southeast, et cetera—thus looks absolutely key to the form that economic growth took in pre-Civil War America.

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