Sketch Solutions to Problem Set 3

1) We are asked to solve for Y when:

- $Y = C+I+G$
- $C = C_0 + C_y Y$

NOTE: When Prof. DeLong asks you to "solve for Y" you are being asked to get Y on one side of the = sign by itself (WITH NOTHING ELSE). The general approach for tackling this kind of problem is:

a) Substitute for everything we can: In this case, we can replace C with $C_0 + C_y Y$
$Y = C_0 + C_y Y + I + G$

b) Move all of the terms containing the variable we are solving for (in this case Y) to the same side of the = sign
$Y - C_y Y = C_0 + I + G$

c) Rearrange to get the variable we are solving for (in this case Y) by itself
$(1-C_y)Y = C_0 + I + G$
$Y=[1/(1-C_y)]\ (C_0 + I + G)$

2) Now we are told that $C_y=2/3$ and asked to figure out how much a $1 **reduction** in investment will reduce Y if G is constant.

a) First, substitute for everything we can: we can replace $C_y = 2/3$
$Y=[1/(1-C_y)]\ (C_0 + I + G)$
$1-C_y = 1 - 2/3 = 1/3$
$1/(1/3) = 3 \rightarrow [1/(1-C_y)] = 3 \rightarrow Y= 3(C_0 + I + G)$

b) Now deal with the question: how does a change in I affect Y when G is constant? Let's write everything in terms of changes (remember Δ means change):
$\Delta Y = 3(\Delta C_0 + \Delta I + \Delta G)$
   - However we are told that G doesn't change, therefore $\Delta G = 0$. C_0 is just a constant so it doesn't change, therefore $\Delta C_0 = 0$.
   - So we have $\Delta Y = 3(0+\Delta I +0) = 3(\Delta I)$
   - How much does Y change when I changes by 1?
     - Well if $\Delta I = 1$, then $\Delta Y =3$

So if I decreases by 1, then Y decreases by 3.

NOTE: Since Prof. Delong tells you that I does not affect G, you could just look at the equation $Y= 3(C_0 + I + G)$ and see that the slope with respect to I is 3 and that therefore a 1 unit change in I will lead to a 3 unit change in Y.

3) We continue assuming that $C_y=2/3$, but now assume that Y effects the amount of Government spending.

a) Once again, substitute for everything we can: using $G=(1/6)Y$
$Y=3(C_0 + I + G) \rightarrow Y=3[C_0 + I + (1/6)Y] \rightarrow Y=3[C_0 + I\ ] + (3/6)Y$

b) Move all of the terms containing the variable we are solving for (in this case Y) to the same side of the = sign
$Y-(1/2)Y = 3[C_0 + I] \rightarrow (1/2)Y=3[C_0 + I]$

c) Rearrange to get the variable we are solving for (in this case Y) by itself
$(1/2)Y=3[C_0 + I] \rightarrow Y=6[C_0 + I]$

d) Now deal with the question: how does a change in I affect Y?

   i. You could just look at the equation in c and see that the coefficient on I is 6 and conclude that a \$1 decrease in investment leads to a \$6 increase in Y (the derivative of Y with respect to I is 6)

   ii. Alternately, you could write this equation in terms of changes. $\Delta Y = 6(\Delta C_0+\Delta I)$. Once again, C_0 is just a constant so it doesn't change, therefore $\Delta C_0 = 0$. $\rightarrow \Delta Y = 6(0+\Delta I) = 6(\Delta I) \rightarrow$ if $\Delta I = 1$, then $\Delta Y = 6$

4) We start by substituting in for the money multiplier m:

$$M = (m_0P^{0.7})H$$

And we then substitute the price level P in for M:

$$P = (m_0P^{0.7})H$$

Divide both sides by $P^{0.7}$:

$$P^{0.3} = (m_0)H$$

And now raise everything to the 3.33 power:

$$P = (m_0H)^{3.33}$$

Now take natural logs:

$$\ln(P) = 3.33\ln(m_0) + 3.33\ln(H)$$

And remember that a change in the natural log is a proportional change. So if $\ln(m_0)$ falls by 0.1—a 10% decline in m_0—then ln(P) falls by .333—a 33.3% decline in P.