Memo: Under What Circumstances Can a Financial Market Learn to Distinguish Good Opinions From Bad Ones?

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I. Introduction

Sociologist Ezra Zuckerman (2004) asks under what circumstances does the "social structural environment typical of financial markets... [have] the necessary features to support the highly sophisticated social learning necessary for incorrect models to valuation to be driven from the market?"

Economists tend to say that markets almost always have the features necessary for them to do so. And, indeed, there is a very powerful argument that if the run is long enough then "noise traders" who trade on bad information or no information at all can be ignored in models of price formation—an argument most forcefully made in Friedman (1953) and Fama (1965). As Friedman puts it, the argument "that speculation is... destabilizing ... is largely equivalent to saying that speculators lose money, since speculation can be destabilizing in general only if speculators on... average sell... low... and buy... high" (Friedman 1953, p. 175). Noise traders thus cannot affect prices too much, and even if they can will not do so for long.

De Long, Shleifer, Summers, and Waldmann (1990) [hereafter DSSW (1990)] partially rebutted this argument within the domain of economic theory. By focusing on some limits of arbitrage, they argued that rational investors who are both risk averse and have short horizons might be unable to do much good in weeding bad information out of financial markets. An important source of risk that must be borne by short-horizon investors engaged in arbitrage against noise traders is the risk that noise traders' beliefs might become even more extreme in the near future. If noise traders today are pessimistic about an asset and have driven down its price, an arbitrageur buying this asset must recognize that in the near future noise traders might become even more pessimistic and drive the price down even further. If the arbitrageur has to liquidate before the price recovers, he suffers a loss. This creates the possibility that noise traders who are on average bullish earn a higher expected return than do risk-averse rational investors with short horizons who engage in arbitrage against noise trading. Noise trader risk drives up returns. If noise traders on average invest more in risky assets, they may earn higher average returns.

Thus if:

1. Rational, sophisticated investors have short horizons.
2. Rational, sophisticated investors are risk averse.
3. Noise traders concentrate their long positions in assets about which their opinions irrationally fluctuate.

It may not be the case that noise traders with incorrect models of valuation lose money, even just relative to rational, sophisticated investors, and so find their influence on the market diminishing over time.

But we can say more. This memo picks up a thread left implicit in DSSW (1990): the interaction between fundamental risk and the possibility that noise traders might flourish in the market. In the domain of economic theory at least, even relatively small increases in fundamental risk curb the enthusiasm of rational investors to bet against noise trading. Such fundamental risk substantially impairs the market's ability to function as a social value calculating mechanism, in the sense that it cannot then be expected to drive incorrect models of valuation from the market. Indeed, as long as conditions 1-3 plus the following are met:

4. New entrants into the market look back at the mean of recent realized returns to decide what valuation strategies to adopt (and ignore the variance).
5. There is sufficient fundamental risk to curb the appetite of sophisticated investors from making long and risky bets against noise trading.

Then, at least as long as we are climbing this particular branch of economic theory, the market does not support the highly sophisticated social learning necessary for incorrect models to valuation to be driven from the market. There are forms of noise trading—incorrect models of valuation—that survive in a stable fashion in the market, no matter how long the run in which it has to learn. There are forms of noise trading that come to dominate the market.

II. Background: The Model of DSSW

The basic model of DSSW (1990) contains two types of investors: noise traders and sophisticated investors. Noise traders falsely believe that they have special information about the future price of the risky asset. They may get their pseudosignals from technical analysts, stock brokers, or economic consultants and irrationally believe that these signals carry information. They select their portfolios on the basis of incorrect beliefs. In response, it is optimal for sophisticated investors to attempt to exploit noise traders' irrational misperceptions by buying when noise traders depress prices and selling when noise traders push prices up.

The model is a stripped down overlapping generations model with two-period lived agents. The only decision agents make is to choose a portfolio when young. The economy contains two assets. One of the assets, the safe asset (s), pays a fixed real dividend r. Asset (s) is in perfectly elastic supply: a unit of it can be created out of and a unit of it turned back into a unit of the consumption good in any period. Thus the price of the safe asset is always fixed at one, and the dividend r paid on asset (s) is the riskless rate of interest.

The other asset, the unsafe asset (u), pays a real dividend d_t^u. But asset (u)'s dividend is risky: each period, it is a constant d* plus an i.i.d. random variable ε with variance σ_ε^2:

(1) $$d_t^u = d^* + \varepsilon_t \qquad \varepsilon_t : N(0,\ \sigma_\varepsilon^2)$$

Moreover, (u) is not in elastic supply: it is in fixed and unchangeable quantity, normalized for simplicity at one unit, with a price in period t of p_t. If the price of each asset were equal to the expected net present value of its future dividends, then assets (u) and (s) would be perfect substitutes and would sell for the same price of one in all periods.

There are two types of investors: sophisticated investors (denoted "i") who have rational expectations and noise traders (denoted "n"). Noise traders are present in measure μ, that sophisticated investors are present in measure 1-μ. Both types of agents choose their portfolios when young to maximize perceived expected utility given their own beliefs about the ex-ante mean of the distribution of the price of (u) at t+1.

A representative sophisticated investor young in period t accurately perceives the distribution of returns from holding the risky asset, and so maximizes expected utility given that distribution.

A representative noise trader young in period t misperceives the expected price of the risky asset next period by an i.i.d. normal random variable ρ_t:

(2) $$E_t^n(p_{t+1}) = E_t^s(p_{t+1}) + \rho_t \quad \rho_t = \rho^* + \eta_t \quad \eta_t : i.i.d.\ N(0,\ \sigma_\eta^2)$$

The mean misperception ρ^* is a measure of the average "bullishness" of the noise traders. The uncertainty η in noise traders' beliefs is uncorrelated across time. Noise traders maximize their own expectation of utility given the next period dividend, the one-period ahead variance of asset prices, and their false belief that the distribution of the price of (u) next period has a mean equal to ρ_t above its true value.

Each agent's utility is a constant absolute risk aversion function of wealth W when old:

(3) $$U = -e^{-2\gamma W}$$

with γ being the coefficient of absolute risk aversion. We assume normally-distributed prices and returns, and so maximizing the expected value of (3) is the same as maximizing:

(4) $$E(W) = -\gamma(V(W))$$

where E(W) is the expectation when young of final wealth, and V(W) is the one-period ahead expected variance of wealth.

III. Solving the Model

From this setup, DSSW (1990) derive sophisticated investor and noise traders demands λ for the risky asset (u):

(5) $$\lambda_t^i = \frac{d^* + E_t p_{t+1} - (1+r)p_t}{2\gamma(\sigma_{p_{t+1}}^2 + \sigma_\varepsilon^2)}$$

(6) $$\lambda_t^n = \frac{d^* + E_t p_{t+1} - (1+r)p_t}{2\gamma(\sigma_{p_{t+1}}^2 + \sigma_\varepsilon^2)} + \frac{\rho^* + \eta_t}{2\gamma(\sigma_{p_{t+1}}^2 + \sigma_\varepsilon^2)}$$

Note that noise traders' and sophisticated investors' demands can be negative; they can take short positions at will. In fact, because returns are unbounded each investor has a positive probability of having negative final wealth. The assumption of normal returns generates great analytical simplicity, but at the cost of allowing consumption to be negative.

Combining these with supply-and-demand:

(7) $$1 = \lambda_t^i + \lambda_t^n$$

and using recursion, DSSW (1990) solve for the price of the risky asset in each period:

(8) $$p_t = \frac{d^*}{r} - \frac{2\gamma\sigma_\varepsilon^2}{r} + \frac{\mu\rho^*}{r} - \frac{2\gamma\mu^2\sigma_\eta^2}{r(1+r)^2} + \frac{\mu\eta_t}{(1+r)}$$

The first term in (8)—d/r—shows the fundamental value of the risky asset would be in a risk-neutral world. The second term— -$2\gamma\sigma_\varepsilon^2/r$ —is a correction for what the price of the risky asset would be in a world in which investors are risk averse but all are rational. The first two terms are the "fundamental" terms in the pricing equation. It is the last three terms that appear in (8) that show the impact on prices of noise traders whose misperceptions are arbitraged by short-horizon risk-averse sophisticated investors.

The fifth term in (8) captures the fluctuations in the price of the risky asset (u) due to the variation of noise traders' misperceptions. Even though the price of asset (u) is not subject to any fundamental uncertainty—all the *fundamental* uncertainty is in the dividend, and cannot be forecast— and is known by a large class of investors, the price p_t nonetheless varies as noise traders' opinions shift. When a generation of noise traders is more "bullish" than the average generation, they bid up the price of (u). When they are more bearish than average, they bid down the price. When they hold their average misperception—when $\eta_t = 0$—this last term is zero. As one would expect, the higher is μ—the more numerous are noise traders relative to sophisticated investors—the more volatile are asset prices.

The third term in (8) captures the deviations of p_t from fundamental values owing to the fact that noise traders are on average “bullish”: this is a price-pressure effect that makes p_t higher than it would otherwise be. Optimistic noise traders bear a greater than average share of risk. Since sophisticated investors bear a smaller share of risk, they require a lower expected excess return and so are willing to pay a higher price for asset (u).

It is the fourth term in (18) that is at the heart of the model. Sophisticated investors must be compensated for bearing the risk that noise traders will become bearish and the price of the risky asset will fall. This effect drives down the price and drives up the return. Noise traders thus “create their own space”: the uncertainty over what next period’s noise traders will believe is a factor giving noise traders’ bullish portfolios higher returns. Any intuition to the effect that investors in the risky asset “ought” to receive higher expected returns because they perform the valuable social function of risk bearing neglects the fact that it is future noise traders’ speculations that are an important source of risk to the current generation of sophisticated investors.

The infinitely-extended overlapping-generations structure of the basic model plays the important role of assuring that each agent's horizon is short. No agent has an opportunity to wait until the price of the risky asset recovers before selling. Such an overlapping-generations structure may be a fruitful way of modelling the effects on prices of a number of institutional features, such as frequent evaluations of money managers’ performance, that may lead even rational, long-lived market participants to care about short-term rather than long-term performance.[1]

IV. Conditions for Noise Traders to Have Higher Expected Returns

The conditions under which noise traders earn higher expected returns than sophisticated investors are straightforward. All agents earn a certain net return of r on their investments in asset (s). The difference between noise traders' and sophisticated investors' total returns is the product of the difference in their holdings of the risky asset (u) and of the excess return paid by a unit of the risky asset (u). Call this difference in returns to the two types of agents ΔR^{n-i}:

$$(9) \qquad \Delta R_t^{n-i} = (\lambda_t^n - \lambda_t^i)(d^* + \varepsilon_{t+1} + p_{t+1} - (1+r)p_t)$$

Substituting:

$$(10) \qquad \Delta R_t^{n-i} = \left(\frac{\rho^* + \eta_t}{2\gamma(\sigma_{p_{t+1}}^2 + \sigma_\varepsilon^2)}\right)(d^* + \varepsilon_{t+1} + p_{t+1} - (1+r)p_t)$$

$$(11) \qquad \Delta R_t^{n-i} = \left(\frac{\rho^* + \eta_t}{2\gamma\left[\sigma_\varepsilon^2 + \frac{\mu_t^2\sigma_\eta^2}{(1+r)^2}\right]}\right)\left(2\gamma\left[\sigma_\varepsilon^2 + \frac{\mu_t^2\sigma_\rho^2}{(1+r)^2}\right] - \mu_t\rho^* - \frac{r\mu_t\eta_t}{(1+r)} + \frac{\mu_t(\eta_{t+1} - \eta_t)}{(1+r)} + \varepsilon_{t+1}\right)$$

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[1] If sophisticated investors' horizons are long relative to the duration of noise traders' optimism or pessimism toward risky assets, sophisticated investors can buy low confident that they will be able to sell high whenever prices revert to the mean—if not next year, the year after; if not the year after, a decade hence; if not a decade hence, half a century hence. Noise trader risk can only become an important deterrent to arbitrage only when the duration of noise traders' misperceptions is of the same order of magnitude or longer than the horizon of sophisticated investors.

(12)

$$\Delta R_t^{n-i} = (\rho^* + \eta_t) - \left(\frac{\mu_t \rho^* (\rho^* + \eta_t)}{2\gamma \left[\sigma_\varepsilon^2 + \frac{\mu_t^2 \sigma_\eta^2}{(1+r)^2} \right]} \right) - \left(\frac{\mu_t \eta_t (\rho^* + \eta_t) r/(1+r)}{2\gamma \left[\sigma_\varepsilon^2 + \frac{\mu_t^2 \sigma_\eta^2}{(1+r)^2} \right]} \right) + \left(\frac{\mu_t (\eta_{t+1} - \eta_t)(\rho^* + \eta_t)/(1+r)}{2\gamma \left[\sigma_\varepsilon^2 + \frac{\mu_t^2 \sigma_\eta^2}{(1+r)^2} \right]} \right) + \left(\frac{\rho^* \varepsilon_{t+1} + \eta_t \varepsilon_{t+1}}{2\gamma \left[\sigma_\varepsilon^2 + \frac{\mu_t^2 \sigma_\eta^2}{(1+r)^2} \right]} \right)$$

The expected value $E(\Delta R^{n-})$ of this is:

$$(13) \qquad E\left(\Delta R_t^{n-i}\right) = \rho^* - \left(\frac{\mu_t \left((\rho^*)^2 + \sigma_\eta^2 \right)}{2\gamma \left[\sigma_\varepsilon^2 + \frac{\mu_t^2 \sigma_\eta^2}{(1+r)^2} \right]} \right)$$

If we set the left-hand side of (13) equal to zero and solve for μ:

$$(14) \qquad 2\gamma \left[\sigma_\varepsilon^2 + \frac{\mu_t^2 \sigma_\eta^2}{(1+r)^2} \right] \rho^* = \mu_t \left((\rho^*)^2 + \sigma_\eta^2 \right)$$

$$(15) \qquad \left(\frac{2\gamma \sigma_\eta^2 \rho^*}{(1+r)^2} \right) \mu_t^2 - \left((\rho^*)^2 + \sigma_\eta^2 \right) \mu_t + 2\gamma \sigma_\varepsilon^2 \rho^* = 0$$

We obtain:

$$(16) \; \mu = \frac{(1+r)^2 \rho^*}{4\gamma \sigma_\eta^2} + \frac{(1+r)^2}{4\gamma \rho^*} \pm \sqrt{ \left(\frac{(1+r)^{2\rho^*}}{4\gamma \sigma_\eta^2} + \frac{(1+r)^2}{4\gamma \rho^*} \right)^2 - \frac{\sigma_\varepsilon^2 (1+r)^2}{\sigma_\eta^2 \rho^*} }$$

If the discriminant of (16) is less than zero, then noise traders' portfolios always have expected returns higher than sophisticated investors.[2] Otherwise, noise traders' portfolios have higher

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[2] This occurs—and noise traders always have higher returns, no matter what the value of μ—when:

expected returns than sophisticated investors' portfolios if:

$$(17)\quad \mu < \mu_1 = \frac{(1+r)^2 \rho *}{4\gamma\sigma_\eta^2} + \frac{(1+r)^2}{4\gamma\rho *} - \sqrt{\left(\frac{(1+r)^{2\rho *}}{4\gamma\sigma_\eta^2} + \frac{(1+r)^2}{4\gamma\rho *}\right)^2 - \frac{\sigma_\varepsilon^2 (1+r)^2}{\sigma_\eta^2 \rho *}}$$

Or:

$$(18)\quad \mu > \mu_2 = \frac{(1+r)^2 \rho *}{4\gamma\sigma_\eta^2} + \frac{(1+r)^2}{4\gamma\rho *} + \sqrt{\left(\frac{(1+r)^{2\rho *}}{4\gamma\sigma_\eta^2} + \frac{(1+r)^2}{4\gamma\rho *}\right)^2 - \frac{\sigma_\varepsilon^2 (1+r)^2}{\sigma_\eta^2 \rho *}}$$

These two equations can be turned into the more convenient:

$$(19)\quad \mu < \mu_1 = (1+r)^2 \left[\frac{\rho *^2 + \sigma_\eta^2}{4\gamma\sigma_\eta^2\rho *}\right]\left[1 - \sqrt{1 - \left(\frac{\sigma_\varepsilon^2}{\sigma_\eta^2\rho *}\right)\left(\frac{\left(4\gamma\sigma_\eta^2\rho *\right)^2}{(1+r)^2\left(\rho *^2 + \sigma_\eta^2\right)^2}\right)}\right]$$

$$(20)\quad \mu > \mu_2 = (1+r)^2 \left[\frac{\rho *^2 + \sigma_\eta^2}{4\gamma\sigma_\eta^2\rho *}\right]\left[1 + \sqrt{1 - \left(\frac{\sigma_\varepsilon^2}{\sigma_\eta^2\rho *}\right)\left(\frac{\left(4\gamma\sigma_\eta^2\rho *\right)^2}{(1+r)^2\left(\rho *^2 + \sigma_\eta^2\right)^2}\right)}\right]$$

Only if neither (19) nor (20) is satisfied will sophisticated investors have higher expected returns from their portfolios than noise traders.[3]

$$(1+r)^2\left(\rho *^2 + \sigma_\eta^2\right)^2 < \sigma_\varepsilon^2\left(16\gamma\sigma_\eta^2\rho *\right).$$

[3] For sufficiently small values of fundamental dividend risk, (19) is approximately:

$$\mu < \mu_1 \approx \frac{(2\gamma)\sigma_\varepsilon^2}{\left(\rho *^2 + \sigma_\eta^2\right)}$$

and (18) is approximately:

How to interpret (19) and (20)? Let us consider some sample parameter values, beginning with the case of no fundamental risk. In that case:

$$(21) \qquad \mu_1 = 0 \;\; and \;\; \mu_2 = (1+r)^2 \left[\frac{\rho*^2 + \sigma_\eta^2}{2\gamma\sigma_\eta^2 \rho*} \right]$$

In the case of no fundamental risk, too small a noise trader share μ guarantees that expected returns will be lower. But if the noise trader share is greater than μ_2 as given in (21), noise trader expected returns will be higher.[4]

Figure 1 plots expected returns as a function of the noise trader share μ for one set of sample parameter values:

Noise trader optimism:	**ρ*=0.3**
Interest rate:	**r=0.5**
Average dividend:	**d*=1**
Variability of beliefs:	**σ_η=0.3**
Fundamental dividend risk:	**σ_ε=0**
Coefficient of risk aversion:	**γ=10**

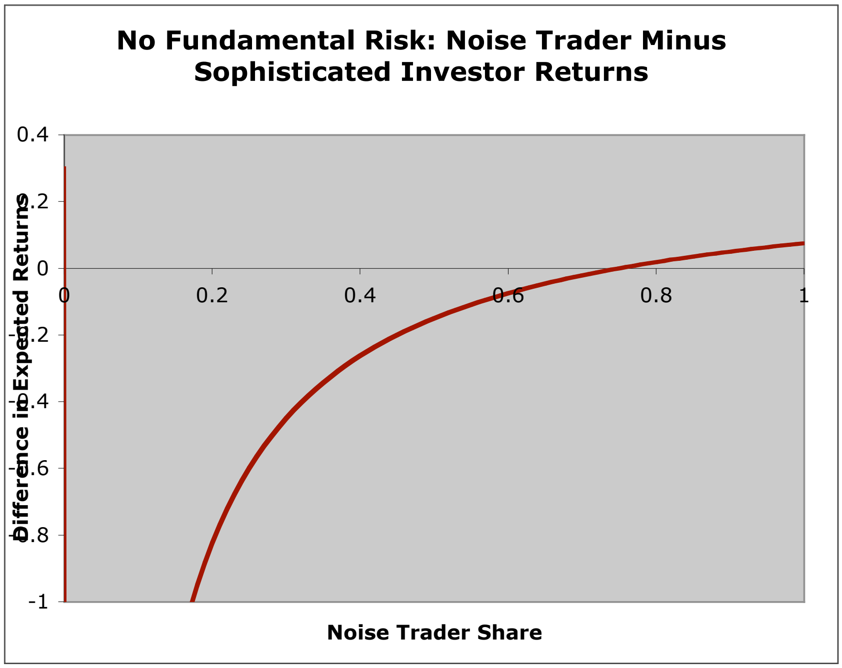
$$\mu > \mu_2 \approx (1+r)^2 \left[\frac{\rho*^2 + \sigma_\eta^2}{2\gamma\sigma_\eta^2 \rho*} \right] - \frac{(2\gamma)\sigma_\varepsilon^2}{\left(\rho*^2 + \sigma_\eta^2\right)}.$$

[4] However, if:

$$2\gamma\sigma_\eta^2\rho* > (1+r)^2\left(\rho*^2 + \sigma_\eta^2\right)$$

the noise trader share would have to be greater than 100% for noise traders to have higher expected returns.

Figure 1



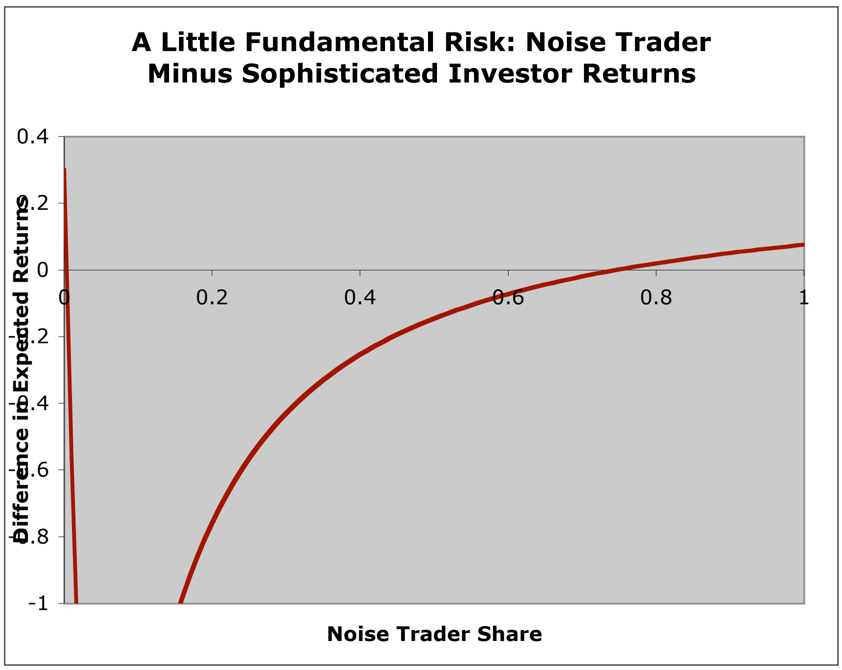
Parameter Values: $\rho^*=0.3$ $r=0.5$ $\sigma_\eta=0.3$ $\sigma_\varepsilon=0$ $\gamma=10$ $\delta^*=1$

When the noise trader share is small, sophisticated investors know that they have nearly a sure thing in betting against the noise traders, and the noise traders think that they have nearly a sure thing in betting on their (wrong) beliefs. As a result, sophisticated investors take large positions short the noise trader view and profit immensely.

As the noise trader share μ rises, the risk of betting against the positions noise traders take grows rapidly. For these parameter values, when $\mu = 0.75$ sophisticated investors are unwilling to bet against noise traders on a sufficient scale. The risk involved in holding asset (u) pushes down its price and pushes up the average return on it. The fact that noise traders hold more of it thus gives them equal returns—even though they do tend to buy high and sell low.

And for values of $\mu > 0.75$, it is the noise trader portfolio that has higher expected returns.

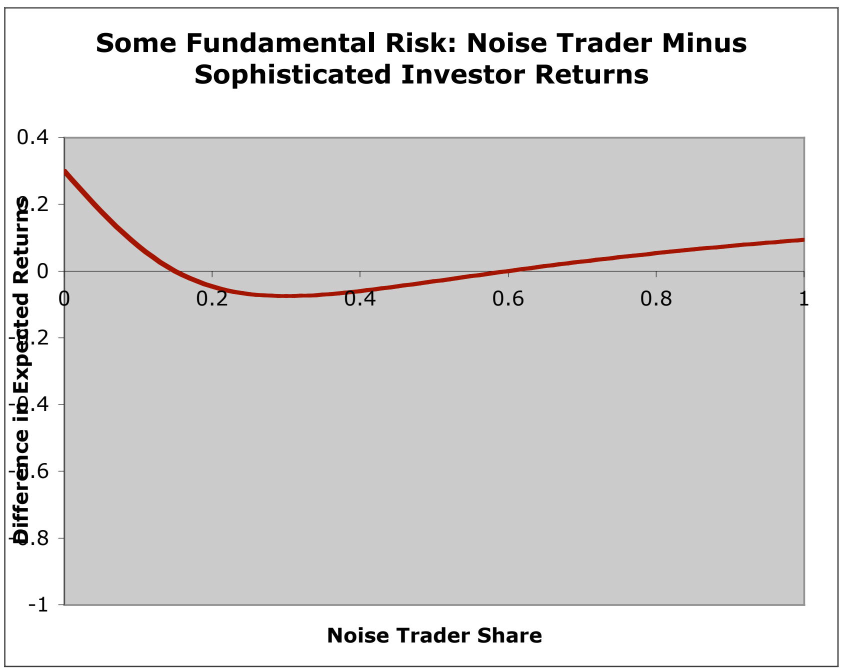
Figure 2



Parameter Values: ρ*=0.3 r=0.5 σ_η=0.3 σ_ε=0.01 γ=10 d*=1

When we add even a little bit of fundamental risk, the dynamics change qualitatively. A very small amount of fundamental risk has next to no effect on the value of μ_2—the value above which noise traders derange prices sufficiently to frighten off sophisticated investors and so earn higher returns. But it does push μ_1 into the positive range, and so creates a small neighborhood of μ=0 in which noise traders have higher returns. An infinitesimal concentration of noise traders do not affect prices and so create no opportunity for sophisticated investors to profit from betting against them. But fundamental risk means that expected returns on the risky asset (u) are higher, and so noise traders' portfolios do better because their optimism leads them to bear greater risk.

Figure 3

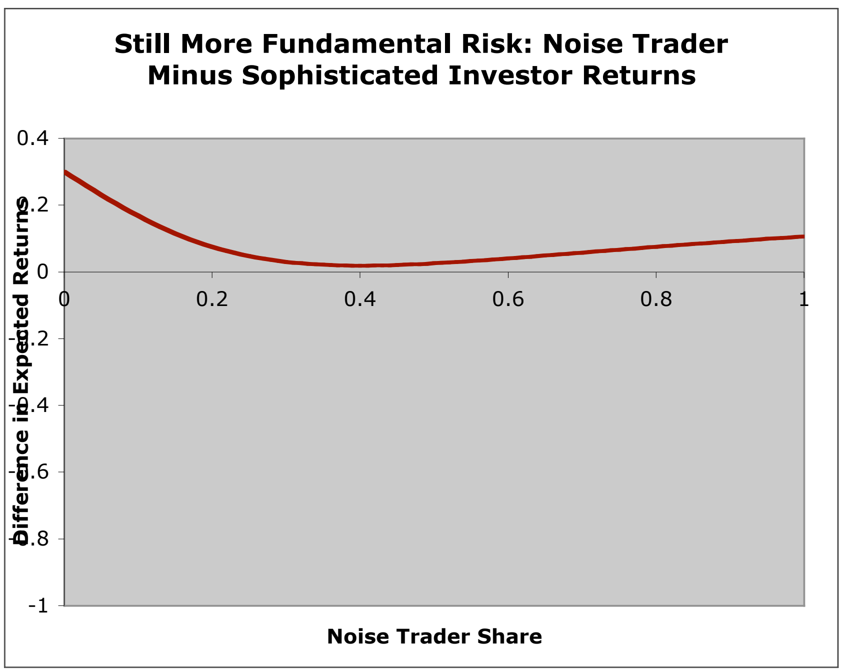


Parameter Values: ρ*=0.3 r=0.5 σ_η=0.3 σ_ε=0.06 γ=10 d*=1

Adding still more fundamental risk brings μ_1 and μ_2 closer together. More fundamental risk enlarges the neighborhood of 0 in which the noise trader share can lie before the ability of sophisticated investors to bet against them reduces relative returns by more than their optimism raises them. More fundamental risk adds another reason for sophisticated investors to be unwilling to bet against noise traders when they are present in sufficient numbers to significantly derange prices.

And when fundamental risk becomes high enough, there are no values of the noise trader share μ for which sophisticated investors earn higher expected returns.[5]

Figure 4



Parameter Values: ρ*=0.3 r=0.3 σ_η=0.3 σ_ε=0.08 γ=10 d*=1

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[5] Which is the case when:

$$(1+r)^2\left(\rho^{*2}+\sigma_\eta^2\right)^2<\sigma_\varepsilon^2\left(16\gamma\sigma_\eta^2\rho^*\right)$$

Higher expected returns of the noise traders come at the cost of holding portfolios with sufficiently higher variance to give noise traders lower expected utility. Since sophisticated investors maximize true expected utility, any trading strategy alternative to theirs that earns a higher mean return must have a variance sufficiently higher to make it unattractive to people who know what they are doing. The fact that bullish noise traders can earn higher returns in the market than sophisticated traders implies that Friedman's simple "market selection" argument is incomplete. Since noise traders' wealth can increase faster than sophisticated investors', it is not possible to make any blanket statement that noise traders lose money and eventually become unimportant. This at least raises the possibility that their importance does not diminish over time.

V. Conditions for Noise Traders to Survive or Dominate in the Long Run

Consider a simple model of the imitation behavior of new generations of investors, who collect information about the performance of the past generation and decide which strategy to follow based on (a) numbers of different types of investors in the past generation, and (b) realized returns—the mean, not the variance—to investors' different portfolio strategies.

In this simple model of imitation, each generation has nearly the same number of investors following noise trader and sophisticated investor strategies as the previous one, but a few investors in each generation change type based on the past relative performance of the two strategies. If noise traders earn a higher return in any period, a fraction of the young who would otherwise have been sophisticated investors become noise traders, and vice versa if noise traders earn a lower return. Moreover, the higher is the difference in realized returns in any period, the more people switch:

(22) $\mu_{t+1} = \mu_t + \zeta \Delta R_t^{n-i}$

Success thus breeds imitation: investment strategies that made their followers richer win converts.

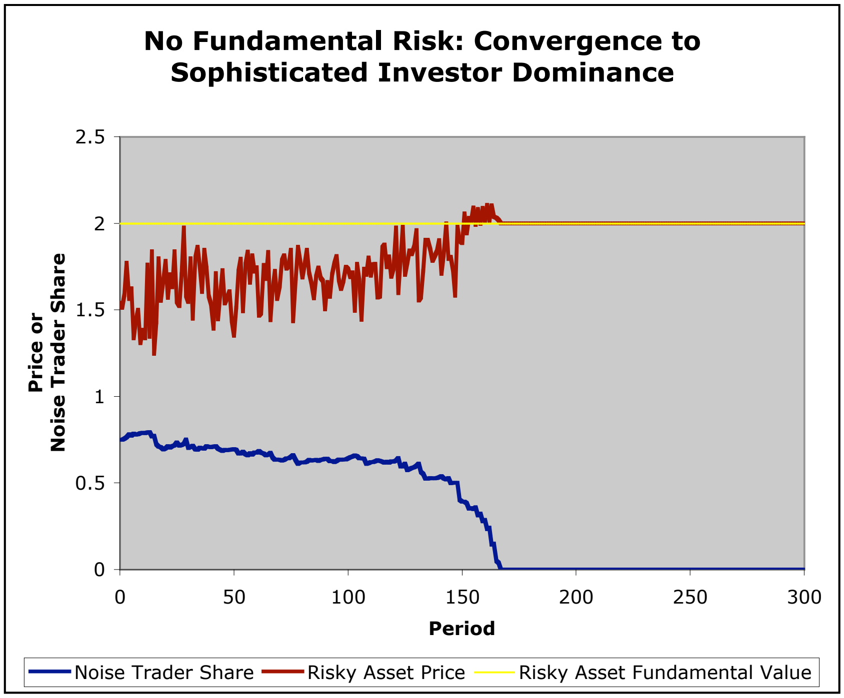
If ζ is not very close to zero, then those investing in any period t have to calculate the effect of the realization of returns on the division of those young in period t+1 between noise traders and sophisticated investors. If ζ is sufficiently small, however, then next period's prices and hence the forward distribution of returns can be calculated under the approximation that the noise trader share will be unchanged. Our Figures 1 through 4 then become not only descriptions of relative expected returns but also descriptions of the dynamics of the noise trader share over time: to the left of μ_1 or to the right of μ_2, the noise trader share grows. In between (if there is an in between) the noise trader share shrinks. For a sufficiently small value of ζ, therefore, the dynamics of the noise trader share μ lead it to one of two places. If the initial share is less than μ_2, the noise trader share converges to μ_1 as given in (19). If the initial share is greater than μ_2, the noise trader share converges to 100% of the investor population.

Of course, these results rely on investors in the incoming generation not being especially clever: if they are clever enough to evaluate not the average returns but the utility achieved by each portfolio strategy, then the long-run dynamics of the noise trader share μ will drive it down to zero.

Figures 5 through 9 exhibit simulation runs of the stochastic dynamics of the noise trader share and the risky asset price for the sample parameter values, under the assumptions (i) that agents are myopic and do not take account of the effects of expected changes in the noise trader share on prices, and (ii) that imitation is based on the difference between noise trader and sophisticated investor portfolios in realized past-period returns, not the difference in utility. It is worth spending a moment looking at such time paths of prices in these simulation runs in order to get a sense of the wide variety of patterns of dynamic behavior that this basic DSSW model can generate. In the model, noise trader beliefs are i.i.d., hence the price series is very choppy—it has effectively no serial correlation. Contrast, in each simulation, the red realized price line with the narrow "fundamental price" line: the yellow line is what the price would be if all the agents in the economy were rational sophisticated investors. The short-term variability of the price series is, of course, proportional to the noise trader share: more noise traders mean more volatility of noise trader demand. And note, across simulations, that the ability of sophisticated investors to damp out the effects of noise trading is never certain, and falls as the amount of fundamental risk in the model rises from zero to a level less than a third of the standard deviation in noise traders' misperceptions.

Figures 5 and 6 consider the case of no fundamental risk. Figure 5 is our first sample simulation run. The initial noise trader share is set at the unstable upper root μ_2, where there is on average neither upward nor downward imitation pressure on the noise trader share. In this simulation run, good luck for the economy quickly pushes the noise trader share down into the range in which sophisticated investors' portfolios have higher returns—and the noise trader share shrinks. The more the noise trader share falls, the smaller are the short-run fluctuations in prices, the less timid are sophisticated investors in their attempts to exploit noise traders' misperceptions, the larger is the return gap, and the faster does the noise trader share fall.

Figure 5



Eventually the noise traders are driven from the market. Thereafter the risky asset's price is constant—and equal to its fundamental value. And thereafter noise traders cannot reenter the market. Because there is no fundamental risk in this simulation run, once the price of the risky asset is constant there is no possibility for the noise trader portfolio to earn higher returns and so attract imitators. In this case—with no fundamental risk—the financial market is able to support the sophisticated social learning process needed to drive the faulty noise trader model of valuation out of the marketplace—although even here it does take quite a while.

Figure 6 shows an alternative case: simulation dynamics when the noise trader share rises into the range in which there is enough noise trading to drive sophisticated investors away from the market. The noise trader share starts out at the unstable upper root

μ_2. Bad luck raises the noise trader share into the range in which noise traders' portfolios earn higher returns. Thereafter the noise trader share grows due to imitation. A rising noise trader share further deranges prices and increases risk. Eventually, noise traders dominate the market, and prices are both significantly and excessively volatile and on average far below true fundamental values. If the noise trader share were ever to drop low enough, the market could learn that their opinions were faulty ones to be exploited, and that the "right" price of the risky asset (u) is the constant yellow fundamental. But the noise trader share doesn't—at least not in historical time.

Figure 6

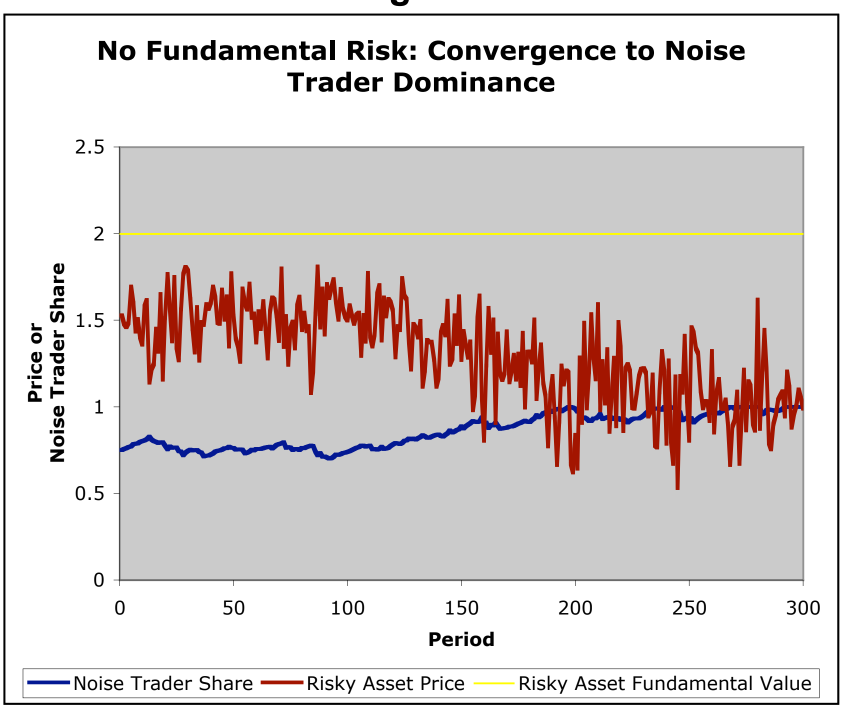


Figure 7 presents a simulation run in which there is a small amount of fundamental risk—σ_ε=0.01. Over time, the noise trader share declines to near zero, as sophisticated investors exploit noise traders and the noise trader share falls. Noise traders, however, repeatedly reenter the market: the existence of systematic risk means that noise traders with their risky asset-heavy portfolios do earn higher expected returns when the noise trader share is very small. Thus whenever the noise trader share hits zero, it bounces back up in a period or two. The market is able to limit and bound the effect of faulty and irrational noise trader models of valuation, but it is not able to completely eliminate their influence.

Figure 7

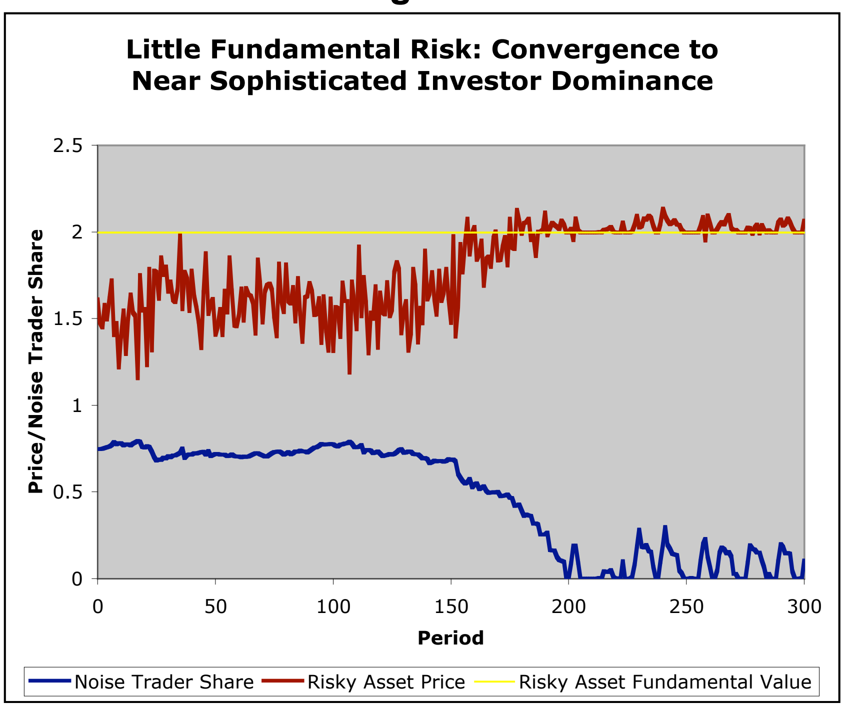
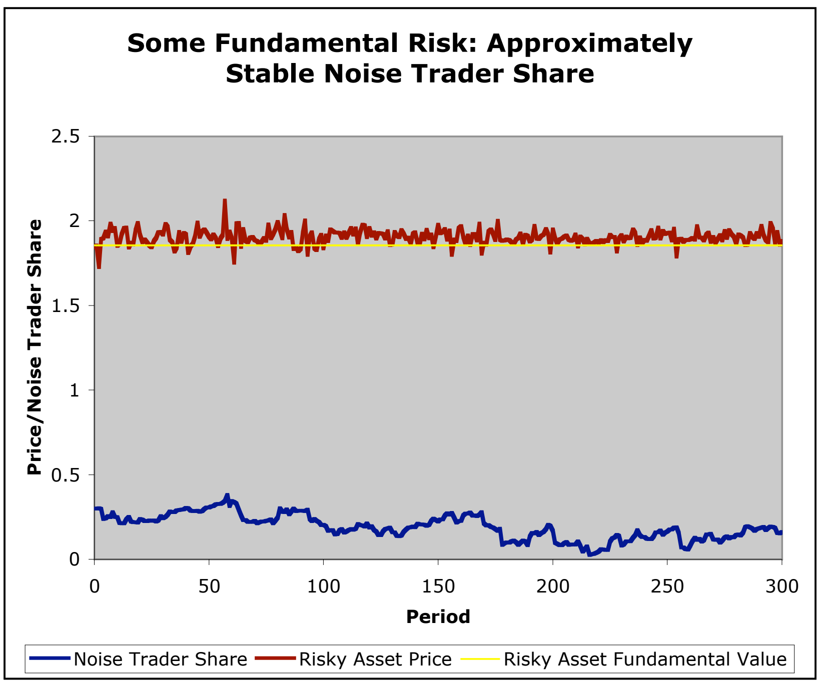


Figure 8 shows a run in which there is enough fundamental risk to push the lower stable root μ_1 up to 0.15: there is enough fundamental risk to give noise trader portfolios an expected return edge whenever the noise trader share is less than 15%. Thus there is on average upward pressure on the noise trader share when it falls below 15%, downward pressure when it rises above 15% (and upward pressure again were it to rise above 60%). In this particular simulation run, however, the noise trader share never rises above 0.4, nor falls to zero. The market is unable to drive the incorrect noise trader valuation out.

Figure 8



With the relatively small noise trader share, asset prices are not terribly volatile. As a result, the upward price pressure that noise traders' optimism exerts outweighs the downward effect on prices of noise trader risk: in this simulation run, risky asset prices are typically above, not below, their fundamental values.

Figure 9

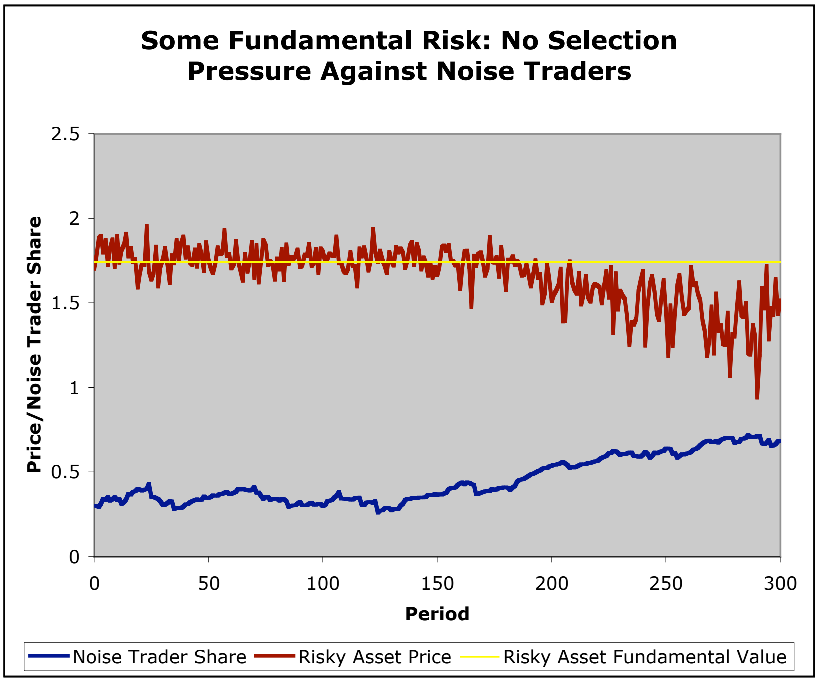


Figure 9 shows a simulation run in which there is enough fundamental risk to eliminate return-based selection pressure against noise traders. No matter what the noise trader share, the expected return on their portfolio is greater than the expected return on sophisticated investors' portfolio. In Figure 9, the share of noise traders climbs slowly and erratically but steadily: eventually it will reach close to 100%.

VI. Conclusion

The principal lesson to draw from this set of results is the fragility of the ability of sophisticated, rational investors with short horizons to curb, let along eliminate, the effect of noise trading. Only when fundamental risk is zero or very small is the market able to keep irrational noise-trader models of valuation on prices from having significant effects on prices, even in the longest run.

It was explicit in DSSW (1990) that the risk created by the unpredictability of unsophisticated investors' opinions can significantly reduce the attractiveness of arbitrage. As long as arbitrageurs have short horizons and so must worry about liquidating their investment in a mispriced asset, their aggressiveness will be limited. Noise trading can thus lead to large divergences between market prices and fundamental values. And noise traders may be compensated for bearing risk, including the risk that they themselves create, and so earn higher returns than sophisticated investors even though they distort prices.

This paper adds to these results: in the case in which the relative abundance of agents' types changes in response to past differences in realized returns, sophisticated investors may well not be able to drive prices to their fundamental values no matter how long the market has to work. The addition of even small amounts of fundamental risk may well guarantee the reverse: that the market will not be able to drive prices to their fundamentals, no matter how long the market has to "learn."

References

DeLong, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann, "Noise Trader Risk in Financial Markets," *Journal of Political Economy* 1990.

Alpert, Mark and Raiffa, Howard. "A Progress Report on the Training of Probability Assessors," pp. 294-305 in Kahneman, Daniel; Slovic, Paul; and Tversky, Amos, eds. *Judgment Under Uncertainty: Heuristics and Biases*. Cambridge, U.K.: Cambridge University Press, 1982.

Black, Fischer. "Noise." *J. Finance* 41 (July 1986): 529-543.

Campbell, John Y., and Kyle, Albert. "Smart Money, Noise Trading, and Stock Price Behavior." Mimeographed. Princeton, NJ: Princeton University, 1987.

Campbell, John Y., and Shiller, Robert. *J.P.E*. 95 (October 1987): 1062-88.

DeLong, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann, "The Size and Incidence of Losses from Noise Trading," *J. Finance* 44 (July 1989): 681-96.

DeLong, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann. "Noise Trader Risk in Financial Markets." *J.P.E.* 1991

Fama, Eugene F. "The Behavior of Stock Market Prices." *J. Business* 38 (1965): 34-105.

Fama, Eugene F., and French, Kenneth. "Permanent and Temporary Components of Stock Prices." *J.P.E.* 96 (April 1988a): 246-73.

Fama, Eugene F., and French, Kenneth. “Dividend Yields and Expected Stock Returns.” *J. Financial Econ.* 24 (October 1988b): 3-26.

Figlewiski, Stephen. “Subjective Information and Market Efficiency in a Betting Market.” *J.P.E.* 87 (February 1979): 75-88.

Friedman, Milton. “The Case For Flexible Exchange Rates.” *Essays in Positive Economics*. Chicago: University of Chicago Press, 1953.

Graham, Benjamin, and Dodd, David. *Security Analysis* . New York: McGraw Hill, 1934.

Grossman, Sanford, and Stiglitz, Joseph E. “On the Impossibility of Informationally Efficient Markets.” *Amer. Econ. Rev*. 70 (June 1980): 393-408.

Hart, Oliver, and Kreps, David. “Price Destabilizing Speculation.” *J.P.E.* 94 (October 1986): 927-952.

Ingram, Beth Fisher. “Equilibrium Modelling of Asset Prices: Rationality v. Rules of Thumb.” Mimeographed. Ithaca, NY: Cornell University, 1987.

Jensen, Michael C. “The Performance of Mutual Funds in the Period 1945-64.” *J. Finance* 23 (May 1968): 389-416.

Keynes, John Maynard. *The General Theory of Employment, Interest and Money* . London: Macmillan, 1936.

King, Braydon. “A Sociology of Market Efficiency” 2004 http://pubsociology.typepad.com/pub/2005/06/a_sociology_of_.html.

Kleidon, Allan W. “Anomalies in Financial Economics.” *J. Bus.* 59

Supplement (December 1986): S285-316.

Mehra, Rajnish, and Prescott, Edward. "The Equity Premium: A Puzzle." *J. Monetary Econ. (*1986).

Shiller, Robert J. "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?" *Amer. Econ. Rev*. 71 (June 1981): 421-36.

Shleifer, Andrei, and Vishny, Robert, "Equilibrium Short Horizons of Investors and Firms." Mimeographed. Chicago, Il.: University of Chicago, 1989.

Stein, Jeremy. "Informational Externalities and Welfare-Reducing Speculation." *J.P.E*. 92 (December 1987): 1123-45.

Zuckerman, Ezra. "Structural Incoherence and Stock Market Activity," *American Sociological Review* 69 (2004): 405-32.