Financial Markets, Noise Traders, and Fundamental Risk: Background Memo

J. Bradford DeLong

U.C. Berkeley and NBER

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Basic Reference: J. Bradford De Long, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann (1990), "Noise Trader Risk in Financial Markets," *Journal of Political Economy*.

Despite the recognition of the abundance in financial markets of "noise traders" who trade on bad information or no information at all, there is a powerful argument that they can be ignored in models of price formation—an argument most forcefully made in Friedman (1953) and Fama (1965). As Friedman puts it, the argument "that speculation is... destabilizing ... is largely equivalent to saying that speculators lose money, since speculation can be destabilizing in general only if speculators on... average sell... low... and buy... high" (Friedman 1953, p. 175). Noise traders thus cannot affect prices too much, and even if they can will not do so for long.

DSSW attempted to partially rebut this argument by focusing on the limits of arbitrage, and arguing that rational investors who are both risk averse and have short horizons might be unable to do much good in weeding bad information out of financial markets. An important source of risk that must be borne by short-horizon investors engaged in arbitrage against noise traders is the risk that noise traders' beliefs might become even more extreme in the near future. If noise traders today are pessimistic about an asset and have driven down its price, an arbitrageur buying this asset must recognize that in the near future noise traders might become even more pessimistic and drive the price down even further. If the arbitrageur has to liquidate before the price recovers, he suffers a loss. This creates the possibility that noise traders who are on average bullish earn a higher expected return than do risk-averse rational investors with short horizons who engage in arbitrage against noise trading. Noise trader risk drives up returns. If noise traders on average invest more in risky assets, they may earn higher average returns. The difference between losing money and losing utility can be an important one.

This background memo picks up a thread that DSSW (1990) barely glanced at: the interaction between fundamental risk and the possibility that noise traders might flourish in the market. It seems likely that further exploration of that model will lead to reasonably strong predictions about the kind of environments in which rational arbitrage will be unable to cleanse financial markets of bad information. The first and most obvious conclusion that needs to be firmed up is that increases in fundamental risk may substantially impair the market's functioning as a social value calculating mechanism. But there will be other conclusions as well…

The basic model of DSSW (1990) contains noise traders and sophisticated investors.

Noise traders falsely believe that they have special information about the future price of the risky asset. They may get their pseudosignals from technical analysts, stock brokers, or economic consultants and irrationally believe that these signals carry information. They select their portfolios on the basis of incorrect beliefs. In response, it is optimal for sophisticated investors to attempt to exploit noise traders' irrational misperceptions by buying when noise traders depress prices and selling when noise traders push prices up.

It is a stripped down overlapping generations model with two-period lived agents. There is no first period consumption, no labor supply decision, no bequest motive: the only decision agents make is to choose a portfolio when young. The economy contains two assets that pay identical expected dividends. One of the assets, the safe asset (s), pays a fixed real dividend r. Asset (s) is in perfectly elastic supply: a unit of it can be created out of and a unit of it turned back into a unit of the consumption good in any period. Thus the price of the safe asset is always fixed at one, and the dividend r paid on asset (s) is the riskless rate of interest.

The other asset, the unsafe asset (u), pays a real dividend d_t^u. But asset (u)'s dividend is risky: each period, it is a constant d* plus an i.i.d. random variable ε with variance σ_ε^2:

(1) $$d_t^u = d^* + \varepsilon_t \qquad \varepsilon_t : N(0,\ \sigma_\varepsilon^2)$$

Moreover, (u) is not in elastic supply: it is in fixed and unchangeable quantity, normalized at one unit, with a price in period t of p_t. If the price of each asset were equal to the expected net present value of its future dividends, then assets (u) and (s) would be perfect substitutes and would sell for the same price of one in all periods.

There are two types of agents: sophisticated investors (denoted "i") who have rational expectations and noise traders (denoted "n"). Noise traders are present in measure μ, that sophisticated investors are present in measure 1-μ. Both types of agents choose their portfolios when young to maximize perceived expected utility given their own beliefs about the ex-ante mean of the distribution of the price of (u) at t+1.

A representative sophisticated investor young in period t accurately perceives the distribution of returns from holding the risky asset, and so maximizes expected utility given that distribution.

A representative noise trader young in period t misperceives the expected price of the risky asset next period by an i.i.d. normal random variable ρ_t:

(2) $$E_t^n(p_{t+1}) = E_t^s(p_{t+1}) + \rho_t \quad \rho_t = \rho^* + \eta_t \quad \eta_t : i.i.d.\ N(0,\ \sigma_\eta^2)$$

The mean misperception ρ^* is a measure of the average "bullishness" of the noise traders. The uncertainty η in noise traders' beliefs is uncorrelated across time. Noise traders maximize their own expectation of utility given the next period dividend, the one-period ahead variance of asset prices, and their false belief that the distribution of the price of (u) next period has a mean equal to ρ_t above its true value.

Each agent's utility is a constant absolute risk aversion function of wealth W when old:

(3) $$U = -e^{-2\gamma W}$$

with γ being the coefficient of absolute risk aversion. We assume normally-distributed prices and returns, and so maximizing the expected value of (3) is the same as maximizing:

(4) $$E(W) = -\gamma(V(W))$$

where E(W) is the expectation when young of final wealth, and V(W) is the one-period ahead expected variance of wealth.

From this setup, DSSW (1990) derive sophisticated investor and noise traders demands λ for the risky asset (u):

(5) $$\lambda_t^i = \frac{d^* + E_t p_{t+1} - (1+r)p_t}{2\gamma(\sigma_{p_{t+1}}^2 + \sigma_\varepsilon^2)}$$

(6) $$\lambda_t^n = \frac{d^* + E_t p_{t+1} - (1+r)p_t}{2\gamma(\sigma_{p_{t+1}}^2 + \sigma_\varepsilon^2)} + \frac{\rho^* + \eta_t}{2\gamma(\sigma_{p_{t+1}}^2 + \sigma_\varepsilon^2)}$$

Note that noise traders' and sophisticated investors' demands can be negative; they can take short positions at will. In fact, because returns are unbounded each investor has a positive probability of having negative final wealth. The assumption of normal returns generates great analytical simplicity, but at the cost of allowing consumption to be negative.

Combining these with supply-and-demand:

(7) $$1 = \lambda_t^i + \lambda_t^n$$

and using recursion, DSSW (1990) solve for the price of the risky asset in each period:

(8) $$p_t = \frac{d*}{r} - \frac{2\gamma\sigma_\varepsilon^2}{r} + \frac{\mu\rho*}{r} - \frac{2\gamma\mu^2\sigma_\eta^2}{r(1+r)^2} + \frac{\mu\eta_t}{(1+r)}$$

The first term in (8)—d/r—shows the fundamental value of the risky asset would be in a risk-neutral world. The second term— $-2\gamma\sigma_\varepsilon^2/r$ —is a correction for what the price of the risky asset would be in a world in which investors are risk averse but all are rational. The first two terms are the "fundamental" terms in the pricing equation. It is the last three terms that appear in (8) that show the impact on prices of noise traders whose misperceptions are arbitraged by short-horizon risk-averse sophisticated investors.

The fifth term in (8) captures the fluctuations in the price of the risky asset (u) due to the variation of noise traders' misperceptions. Even though the price of asset (u) is not subject to any fundamental uncertainty—all the *fundamental* uncertainty is in the dividend, and cannot be forecast— and is known by a large class of investors, the price p_t nonetheless varies as noise traders' opinions shift. When a generation of noise traders is more "bullish" than the average generation, they bid up the price of (u). When they are more bearish than average, they bid down the price. When they hold their average misperception—when $\eta_t = 0$—this last term is zero. As one would expect, the higher is μ—the more numerous are noise traders relative to sophisticated investors—the more volatile are asset prices.

The third term in (8) captures the deviations of p_t from fundamental values owing to the fact that noise traders are on average "bullish": this is a price-pressure effect that makes p_t higher than it would otherwise be. Optimistic noise traders bear a greater than average share of risk. Since sophisticated investors bear a smaller share of risk, they require a lower expected excess return and so are willing to pay a higher price for asset (u).

It is the fourth term in (18) that is at the heart of the model. Sophisticated investors must be compensated for bearing the risk that noise traders will become bearish and the price of the risky asset will fall. This effect drives down the price and drives up the return. Noise traders thus "create their own space": the uncertainty over what next period's noise traders will believe is a factor giving noise traders' bullish portfolios higher returns. Any intuition to the effect that investors in the risky asset "ought" to receive higher expected returns because they perform the valuable social function of risk bearing neglects the fact that it is future noise traders' speculations that are an important source of risk to the current generation of sophisticated investors.

The infinitely-extended overlapping-generations structure of the basic model plays the important role of assuring that each agent's horizon is short. No agent has an opportunity to wait until the price of the risky asset recovers before selling. Such an overlapping-generations structure may be a fruitful way of modelling the effects on prices of a number of institutional features, such as frequent evaluations of money managers' performance, that may lead even rational, long-lived market participants to care about short-term rather than long-term performance.[1]

The conditions under which noise traders earn higher expected returns than sophisticated investors are straightforward. All agents earn a certain net return of r on their investments in asset (s). The difference between noise traders' and sophisticated investors' total returns is the product of the difference in their holdings of the risky asset (u) and of the excess return paid by a unit of the risky asset (u). Call this difference in returns to the two types of agents ΔR^{n-i}:

$$\text{(9)} \qquad \Delta R_t^{n-i} = (\lambda_t^n - \lambda_t^i)(d^* + \varepsilon_{t+1} + p_{t+1} - (1+r)p_t)$$

Substituting:

[1] If sophisticated investors' horizons are long relative to the duration of noise traders' optimism or pessimism toward risky assets, sophisticated investors can buy low confident that they will be able to sell high whenever prices revert to the mean—if not next year, the year after; if not the year after, a decade hence; if not a decade hence, half a century hence. Noise trader risk can only become an important deterrent to arbitrage only when the duration of noise traders' misperceptions is of the same order of magnitude or longer than the horizon of sophisticated investors.

(10) $$\Delta R_t^{n-i} = \left(\frac{\rho^* + \eta_t}{2\gamma(\sigma_{p_{t+1}}^2 + \sigma_\varepsilon^2)} \right) (d^* + \varepsilon_{t+1} + p_{t+1} - (1+r)p_t)$$

(11) $$\Delta R_t^{n-i} = \left(\frac{\rho^* + \eta_t}{2\gamma\left[\sigma_\varepsilon^2 + \frac{\mu_t^2 \sigma_\eta^2}{(1+r)^2}\right]} \right) \left(2\gamma\left[\sigma_\varepsilon^2 + \frac{\mu_t^2 \sigma_\rho^2}{(1+r)^2}\right] - \mu_t \rho^* - \frac{r\mu_t \eta_t}{(1+r)} + \frac{\mu_t(\eta_{t+1} - \eta_t)}{(1+r)} + \varepsilon_{t+1} \right)$$

(12) $$\Delta R_t^{n-i} = (\rho^* + \eta_t) - \left(\frac{\mu_t \rho^* (\rho^* + \eta_t)}{2\gamma\left[\sigma_\varepsilon^2 + \frac{\mu_t^2 \sigma_\eta^2}{(1+r)^2}\right]} \right) - \left(\frac{\mu_t \eta_t (\rho^* + \eta_t) r/(1+r)}{2\gamma\left[\sigma_\varepsilon^2 + \frac{\mu_t^2 \sigma_\eta^2}{(1+r)^2}\right]} \right) + \left(\frac{\mu_t(\eta_{t+1} - \eta_t)(\rho^* + \eta_t)/(1+r)}{2\gamma\left[\sigma_\varepsilon^2 + \frac{\mu_t^2 \sigma_\eta^2}{(1+r)^2}\right]} \right) + \left(\frac{\rho^* \varepsilon_{t+1} + \eta_t \varepsilon_{t+1}}{2\gamma\left[\sigma_\varepsilon^2 + \frac{\mu_t^2 \sigma_\eta^2}{(1+r)^2}\right]} \right)$$

The expected value $E(\Delta R^{n-})$ of this is:

(13) $$E\left(\Delta R_t^{n-i}\right) = \rho^* - \left(\frac{\mu_t\left((\rho^*)^2 + \sigma_\eta^2\right)}{2\gamma\left[\sigma_\varepsilon^2 + \frac{\mu_t^2 \sigma_\eta^2}{(1+r)^2}\right]} \right)$$

If we set the left-hand side of (13) equal to zero and solve for μ:

(14) $$2\gamma\left[\sigma_\varepsilon^2 + \frac{\mu_t^2 \sigma_\eta^2}{(1+r)^2}\right]\rho^* = \mu_t\left((\rho^*)^2 + \sigma_\eta^2\right)$$

(15) $$\left(\frac{2\gamma\sigma_\eta^2 \rho^*}{(1+r)^2} \right)\mu_t^2 - \left((\rho^*)^2 + \sigma_\eta^2\right)\mu_t + 2\gamma\sigma_\varepsilon^2 \rho^* = 0$$

We obtain:

$$\text{(16)} \qquad \mu = \frac{(1+r)^2 \rho *}{4\gamma\sigma_\eta^2} + \frac{(1+r)^2}{4\gamma\rho *} \pm \sqrt{\left(\frac{(1+r)^{2\rho *}}{4\gamma\sigma_\eta^2} + \frac{(1+r)^2}{4\gamma\rho *}\right)^2 - \frac{\sigma_\varepsilon^2 (1+r)^2}{\sigma_\eta^2 \rho *}}$$

If the discriminant of (16) is less than zero, then noise traders' portfolios always have expected returns higher than sophisticated investors.[2] Otherwise, noise traders' portfolios have higher expected returns than sophisticated investors' portfolios if:

$$\text{(17)} \qquad \mu < \mu_1 = \frac{(1+r)^2 \rho *}{4\gamma\sigma_\eta^2} + \frac{(1+r)^2}{4\gamma\rho *} - \sqrt{\left(\frac{(1+r)^{2\rho *}}{4\gamma\sigma_\eta^2} + \frac{(1+r)^2}{4\gamma\rho *}\right)^2 - \frac{\sigma_\varepsilon^2 (1+r)^2}{\sigma_\eta^2 \rho *}}$$

Or:

$$\text{(18)} \quad \mu > \mu_2 = \frac{(1+r)^2 \rho *}{4\gamma\sigma_\eta^2} + \frac{(1+r)^2}{4\gamma\rho *} + \sqrt{\left(\frac{(1+r)^{2\rho *}}{4\gamma\sigma_\eta^2} + \frac{(1+r)^2}{4\gamma\rho *}\right)^2 - \frac{\sigma_\varepsilon^2 (1+r)^2}{\sigma_\eta^2 \rho *}}$$

These two equations can be turned into the more convenient:

[2] This occurs—and noise traders always have higher returns, no matter what the value of μ—when:

$$(1+r)^2\left(\rho *^2 + \sigma_\eta^2\right)^2 < \sigma_\varepsilon^2\left(16\gamma\sigma_\eta^2\rho *\right).$$

$$\text{(19)} \qquad \mu < \mu_1 = (1+r)^2 \left[\frac{\rho *^2 + \sigma_\eta^2}{4\gamma\sigma_\eta^2 \rho *} \right] \left[1 - \sqrt{1 - \left(\frac{\sigma_\varepsilon^2}{\sigma_\eta^2 \rho *} \right) \left(\frac{\left(4\gamma\sigma_\eta^2 \rho *\right)^2}{(1+r)^2 \left(\rho *^2 + \sigma_\eta^2\right)^2} \right)} \right]$$

$$\text{(20)} \qquad \mu > \mu_2 = (1+r)^2 \left[\frac{\rho *^2 + \sigma_\eta^2}{4\gamma\sigma_\eta^2 \rho *} \right] \left[1 + \sqrt{1 - \left(\frac{\sigma_\varepsilon^2}{\sigma_\eta^2 \rho *} \right) \left(\frac{\left(4\gamma\sigma_\eta^2 \rho *\right)^2}{(1+r)^2 \left(\rho *^2 + \sigma_\eta^2\right)^2} \right)} \right]$$

Only if neither (19) nor (20) is satisfied will sophisticated investors have higher expected returns from their portfolios than noise traders.[3]

How to interpret (19) and (20)? Let us consider some sample parameter values, beginning with the case of no fundamental risk. In that case:

$$\text{(21)} \qquad \mu_1 = 0 \;\; and \;\; \mu_2 = (1+r)^2 \left[\frac{\rho *^2 + \sigma_\eta^2}{2\gamma\sigma_\eta^2 \rho *} \right]$$

In the case of no fundamental risk, too small a noise trader share μ guarantees that expected returns will be lower. But if the noise trader share is greater than μ_2 as given in

[3] For sufficiently small values of fundamental dividend risk, (19) is approximately:

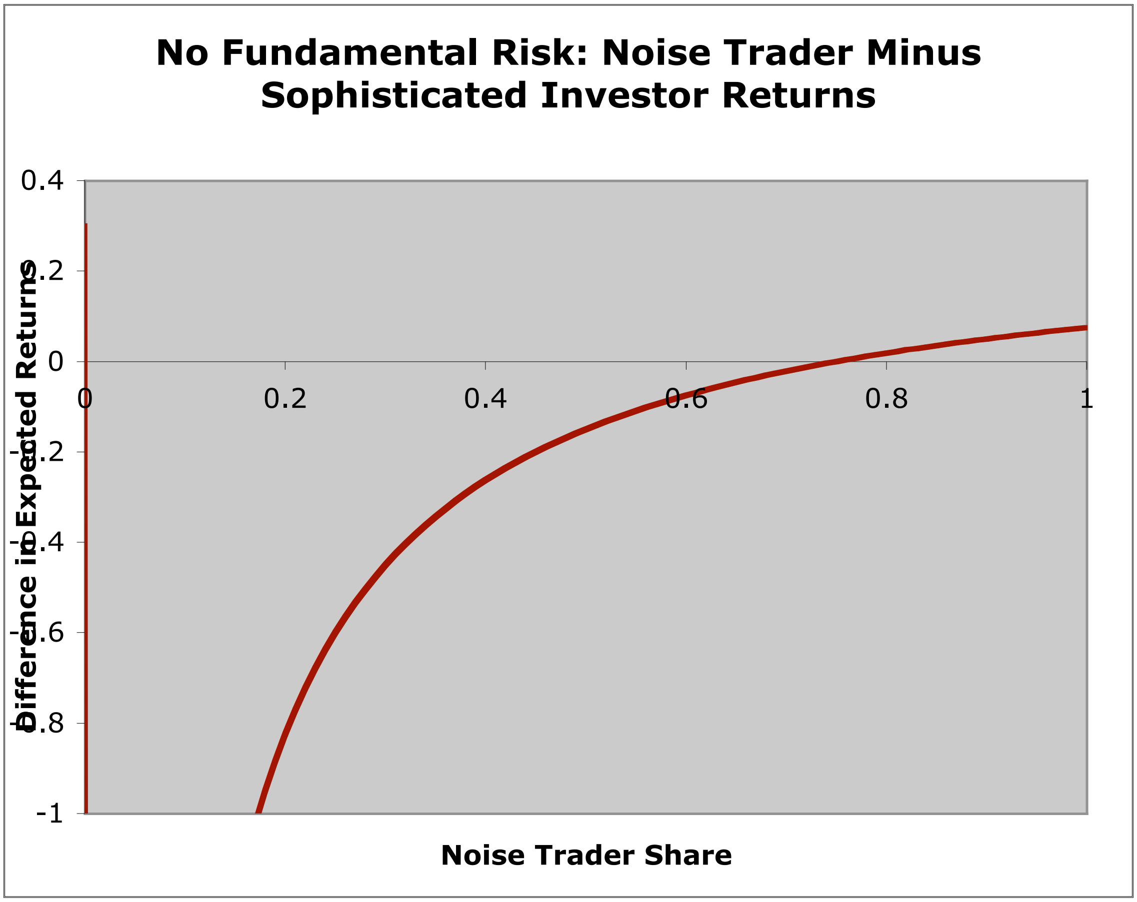
$$\mu < \mu_1 \approx \frac{(2\gamma)\sigma_\varepsilon^2}{\left(\rho *^2 + \sigma_\eta^2\right)}$$

and (18) is approximately:

$$\mu > \mu_2 \approx (1+r)^2 \left[\frac{\rho *^2 + \sigma_\eta^2}{2\gamma\sigma_\eta^2 \rho *} \right] - \frac{(2\gamma)\sigma_\varepsilon^2}{\left(\rho *^2 + \sigma_\eta^2\right)}.$$

(21), noise trader expected returns will be higher.[4]

Figure 1



Parameter Values: $\rho*=0.3$ $r=0.5$ $\sigma_\eta=0.3$ $\sigma_\varepsilon=0$ $\gamma=10$ $\delta*=1$

Figure 1 plots expected returns as a function of the noise trader share μ for one set of sample parameter values:

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[4] However, if:

$$2\gamma\sigma_\eta^2\rho* > (1+r)^2\left(\rho*^2 + \sigma_\eta^2\right)$$

the noise trader share would have to be greater than 100% for noise traders to have higher expected returns.

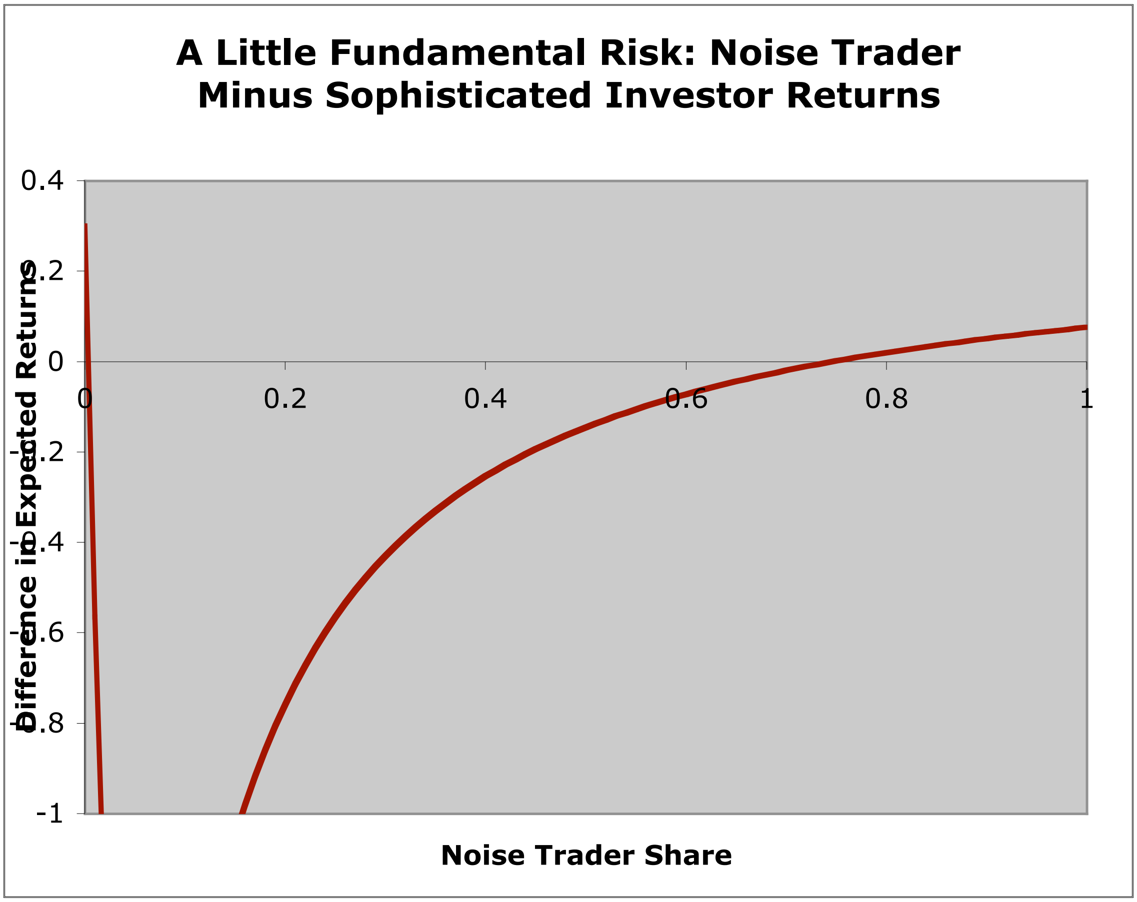
Noise trader optimism:	**ρ*=0.3**
Interest rate:	**r=0.5**
Average dividend:	**d*=1**
Variability of beliefs:	**σ_η=0.3**
Fundamental dividend risk:	**σ_ε=0**
Coefficient of risk aversion:	**γ=10**

When the noise trader share is small, sophisticated investors know that they have nearly a sure thing in betting against the noise traders, and the noise traders think that they have nearly a sure thing in betting on their (wrong) beliefs. As a result, sophisticated investors take large positions short the noise trader view and profit immensely.

As the noise trader share μ rises, the risk of betting against the positions noise traders take grows rapidly. For these parameter values, when $\mu = 0.75$ sophisticated investors are unwilling to bet against noise traders on a sufficient scale. The risk involved in holding asset (u) pushes down its price and pushes up the average return on it. The fact that noise traders hold more of it thus gives them equal returns—even though they do tend to buy high and sell low.

And for values of $\mu > 0.75$, it is the noise trader portfolio that has higher expected returns.

Figure 2

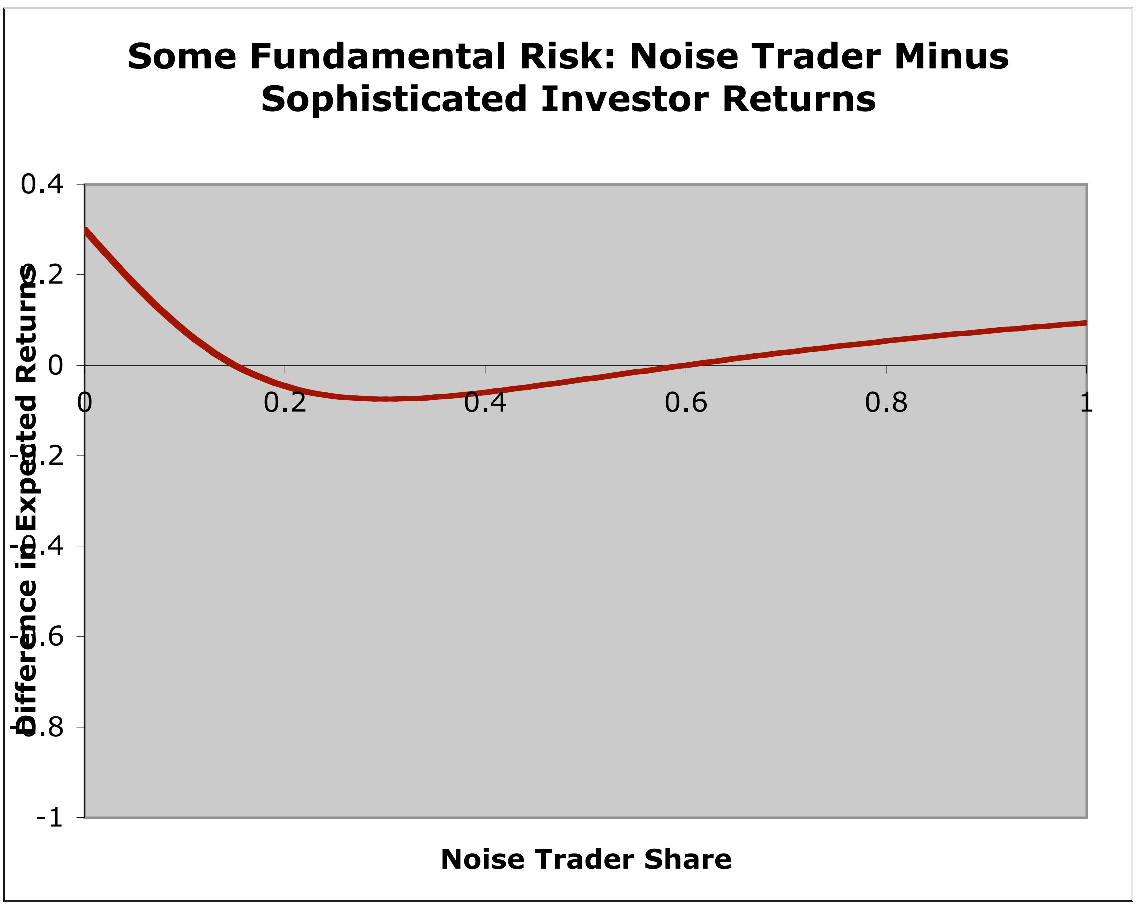


Parameter Values: $\rho*=0.3$ $r=0.5$ $\sigma_\eta=0.3$ $\sigma_\varepsilon=0.01$ $\gamma=10$ $d*=1$

When we add even a little bit of fundamental risk, the dynamics change qualitatively. A very small amount of fundamental risk has next to no effect on the value of μ_2—the value above which noise traders derange prices sufficiently to frighten off sophisticated investors and so earn higher returns. But it does push μ_1 into the positive range, and so creates a small neighborhood of $\mu=0$ in which noise traders have higher returns. An infinitesimal concentration of noise traders do not affect prices and so create no opportunity for sophisticated investors to profit from betting against them. But fundamental risk means that expected returns on the risky asset (u) are higher, and so noise traders' portfolios do better because their optimism leads them to bear greater risk.

Adding still more fundamental risk brings μ_1 and μ_2 closer together. More fundamental risk enlarges the neighborhood of 0 in which the noise trader share can lie before the ability of sophisticated investors to bet against them reduces relative returns by more than their optimism raises them. More fundamental risk adds another reason for sophisticated investors to be unwilling to bet against noise traders when they are present in sufficient numbers to significantly derange prices.

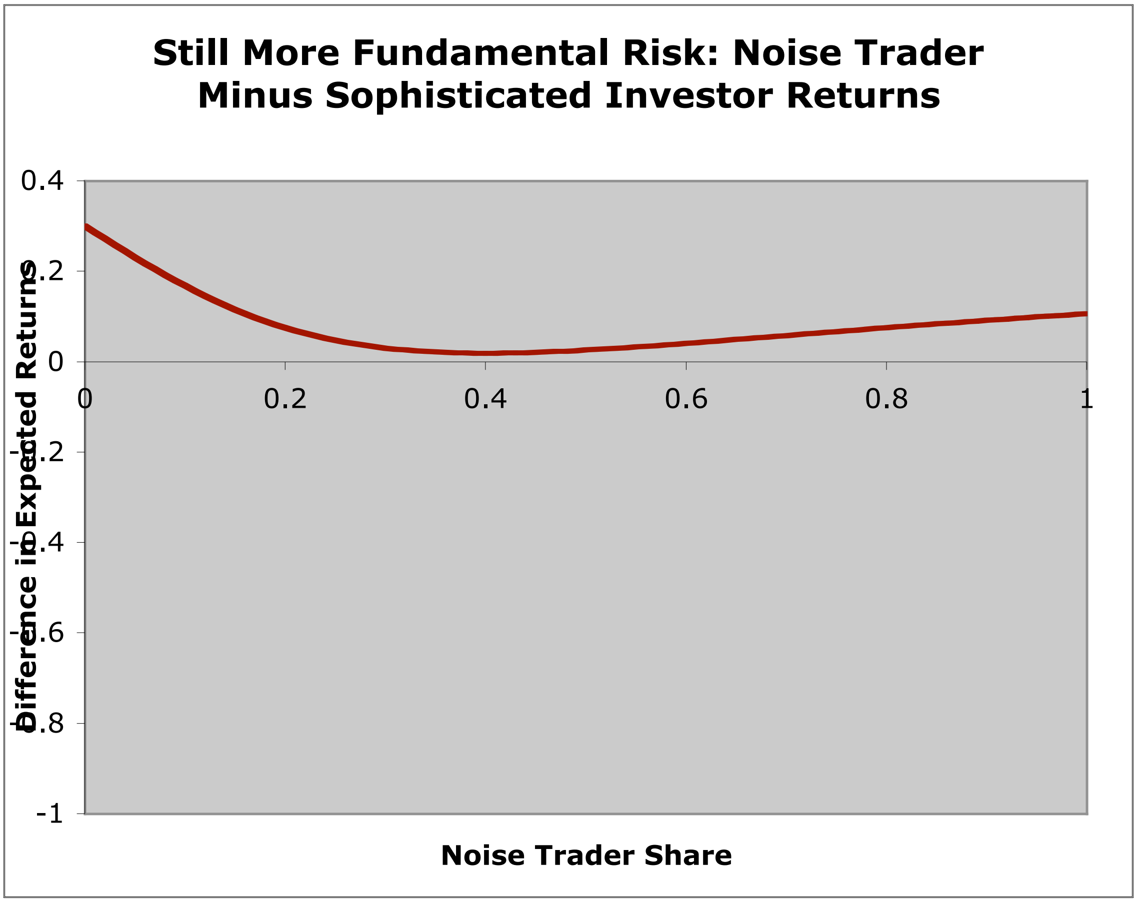
Figure 3



Parameter Values: ρ*=0.3 r=0.5 σ_η=0.3 σ_ε=0.06 γ=10 d*=1

And when fundamental risk becomes high enough, there are no values of the noise trader share μ for which sophisticated investors earn higher expected returns.[5]

Figure 4



Parameter Values: $\rho*=0.3$ r=0.3 $\sigma_\eta=0.3$ $\sigma_\varepsilon=0.08$ $\gamma=10$ d*=1

Higher expected returns of the noise traders come at the cost of holding portfolios with sufficiently higher variance to give noise traders lower expected utility. Since sophisticated investors maximize true expected utility, any trading strategy alternative to

[5] Which is the case when:

$$(1+r)^2\left(\rho*^2+\sigma_\eta^2\right)^2<\sigma_\varepsilon^2\left(16\gamma\sigma_\eta^2\rho*\right)$$

theirs that earns a higher mean return must have a variance sufficiently higher to make it unattractive to people who know what they are doing. The fact that bullish noise traders can earn higher returns in the market than sophisticated traders implies that Friedman's simple "market selection" argument is incomplete. Since noise traders' wealth can increase faster than sophisticated investors', it is not possible to make any blanket statement that noise traders lose money and eventually become unimportant. This at least raises the possibility that their importance does not diminish over time.

Our two-period model does not permit us to examine the accumulation of wealth by noise traders. As an alternative consider the emulative behavior of new generations of Investors, who collect information about the performance of the past generation and decide which strategy to follow. Each generation thus has nearly the same number of investors following noise trader and sophisticated investor strategies as the previous one, but a few investors in each generation change type based on the past relative performance of the two strategies. If noise traders earn a higher return in any period, a fraction of the young who would otherwise have been sophisticated investors become noise traders, and vice versa if noise traders earn a lower return. Moreover, the higher is the difference in realized returns in any period, the more people switch:

(22) $$\mu_{t+1} = \mu_t + \zeta \Delta R_t^{n-i}$$

Success breeds imitation: investment strategies that made their followers richer win converts.

If ζ is not very close to zero, then those investing in any period t have to calculate the effect of the realization of returns on the division of those young in period t+1 between noise traders and sophisticated investors. If ζ is sufficiently small, however, then next period's prices and hence the forward distribution of returns can be calculated under the approximation that the noise trader share will be unchanged. Our Figures 1 through 4 then become not only descriptions of relative expected returns but also descriptions of the dynamics of the noise trader share over time: to the left of μ_1 or to the right of μ_2, the noise trader share grows. In between (if there is an in between) the noise trader share shrinks. For a sufficiently small value of ζ, therefore, the dynamics of the noise trader share μ lead it to one of two places. If the initial share is less than μ_2, the noise trader share converges to μ_1 as given in (19). If the initial share is greater than μ_2, the noise trader share converges to 100% of the investor population.

Of course, these results rely on investors in the incoming generation not being especially clever: if they are clever enough to evaluate not the average returns but the utility achieved by each portfolio strategy, then the long-run dynamics of the noise trader share μ will drive it down to zero.

Figures 5 through 9 exhibit simulation runs of the stochastic dynamics of the noise trader share and the risky asset price for the sample parameter values, under the assumption that agents are myopic and do not take account of the effects of expected changes in the noise trader share on prices. Figures 5 and 6 consider the case of no fundamental risk. Figure 5 shows a simulation run in which the noise trader share quickly falls into the range in which there is insufficient noise trading to drive sophisticated investors away from the market, and in which over time sophisticated investors exploit noise traders, the noise trader share falls, and eventually noise traders are driven from the market. Thereafter noise traders do not reenter: since there is no fundamental risk in this simulation run, once the price of the risky asset is constant there is no noise trader portfolio to earn higher returns.

Figure 5

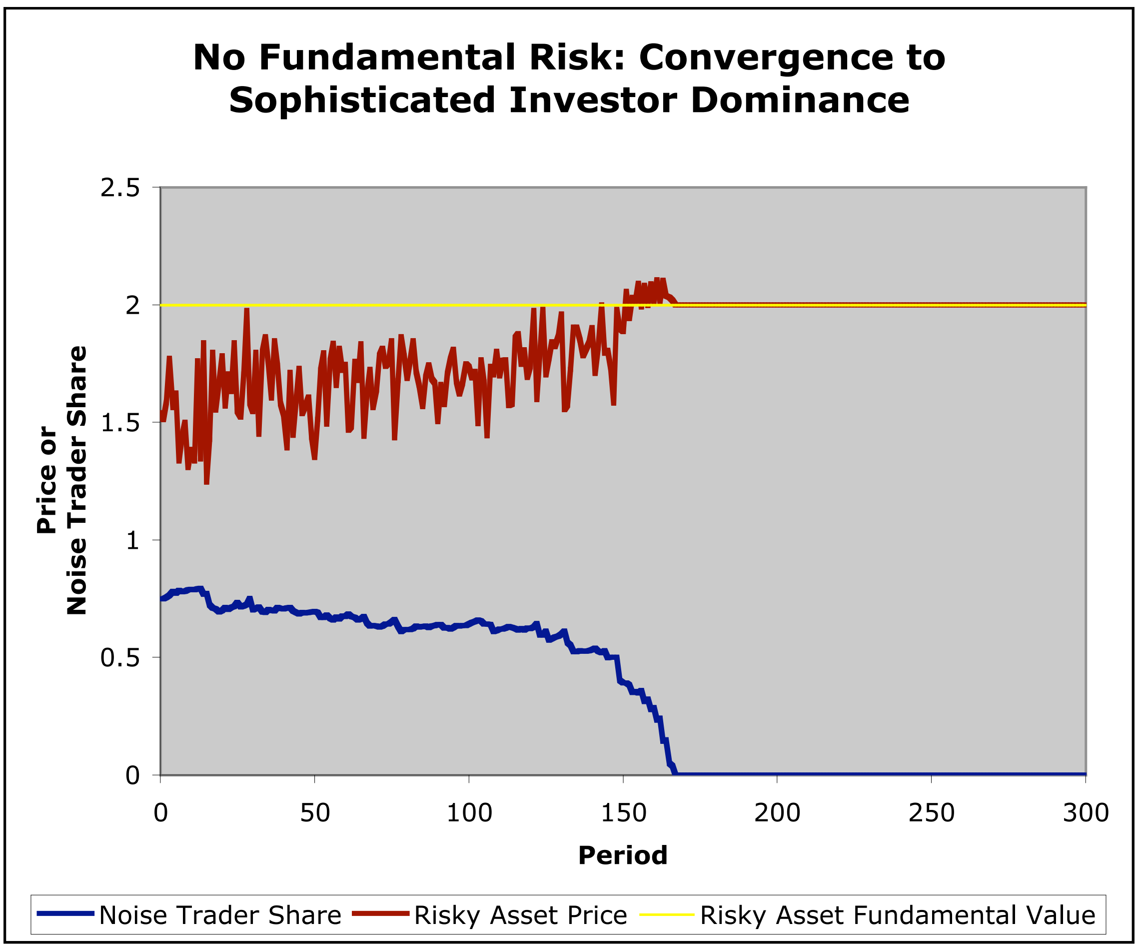


Figure 6 shows dynamics when the noise trader share rises into the range in which there is enough noise trading to drive sophisticated investors away from the market. Noise traders' portfolios earn higher returns. Over time, the noise trader share grows due to imitation. A rising noise trader share further deranges prices and increases risk. Eventually, noise traders dominate market.

Figure 6

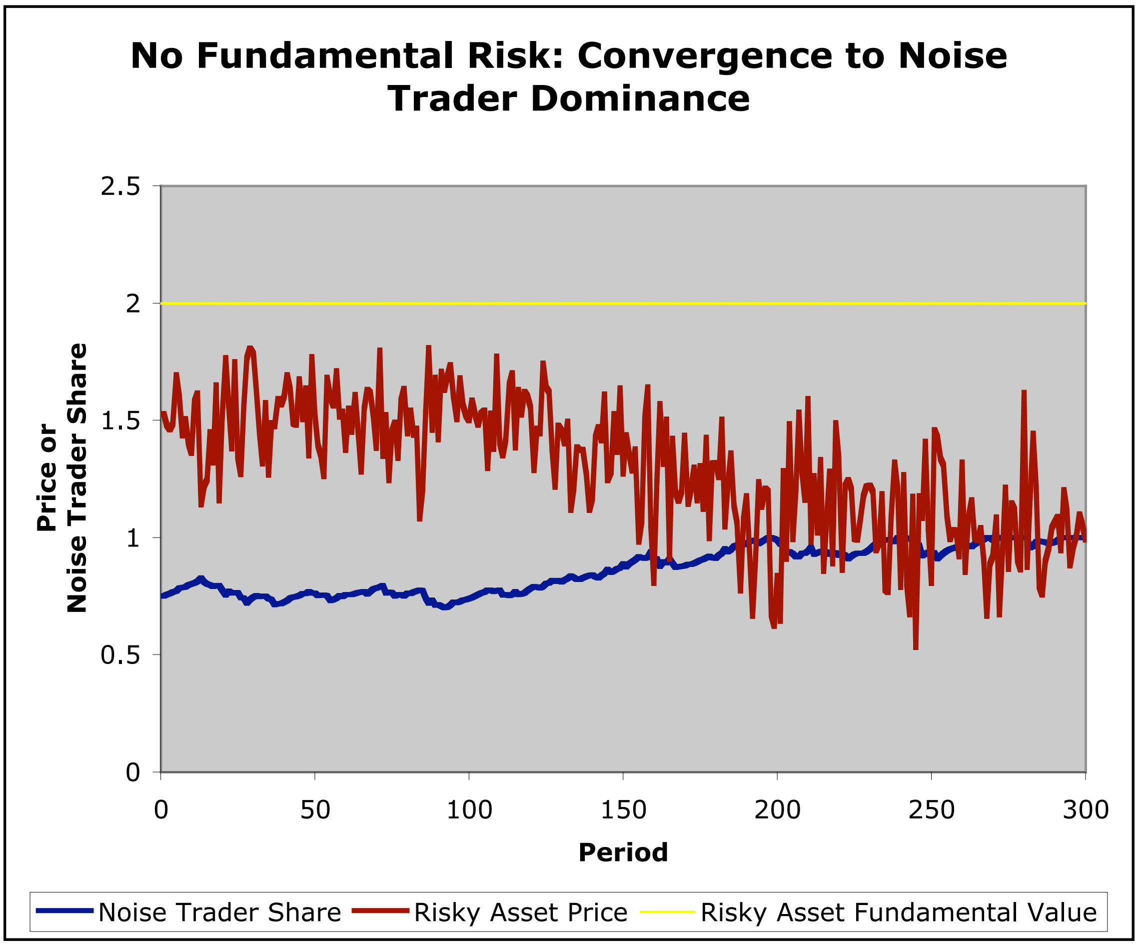


Figure 7 presents a simulation run in which there is a small amount of fundamental risk—σ_ε=0.01. Over time, the noise trader share declines to near zero, as sophisticated investors exploit noise traders and the noise trader share falls. Even when noise traders are eliminated, however, they are able to reenter the market: the existence of systematic risk means that noise traders with their risky asset-heavy portfolios do earn higher expected returns when the noise trader share is very small. Thus whenever the noise trader share hits zero, it bounces back up.

Figure 7

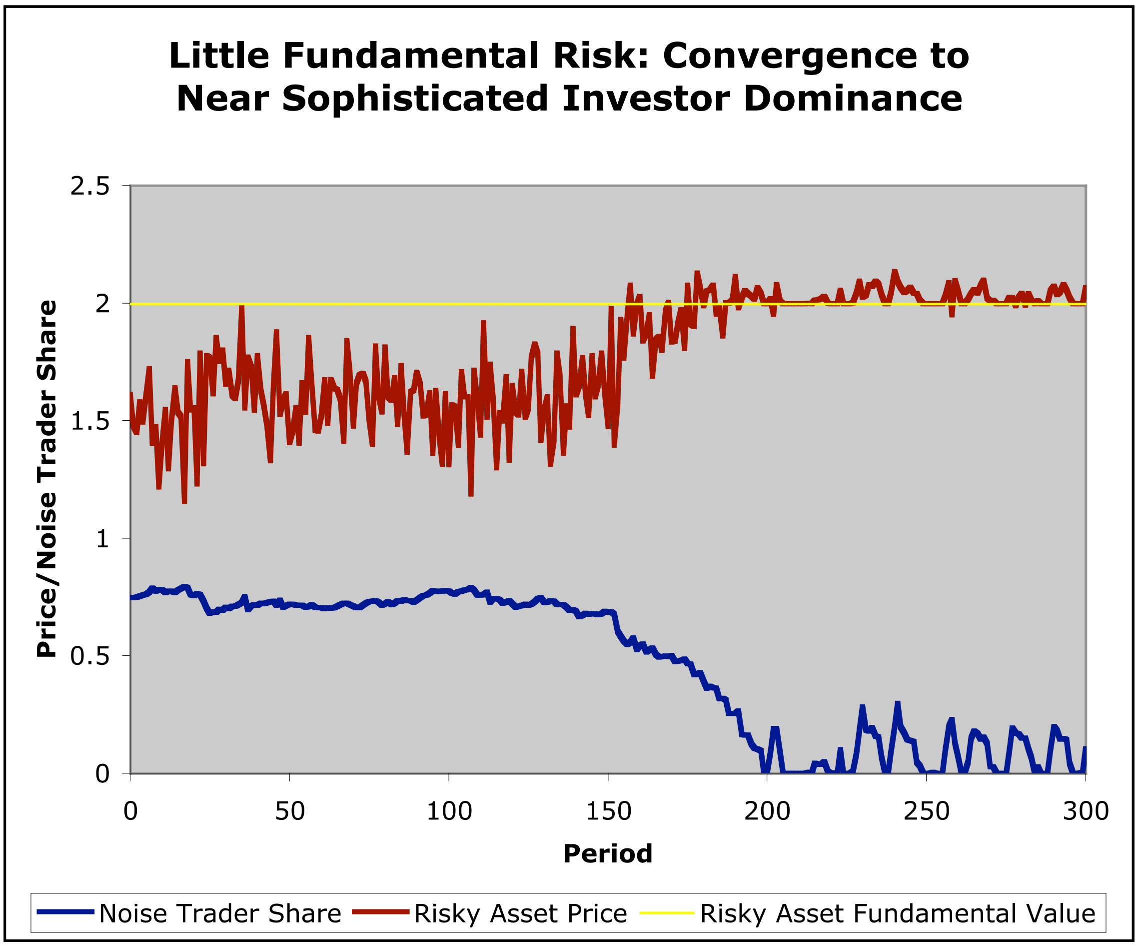


Figure 8 shows a run in which the root μ_1 is equal to 0.15: because there is enough fundamental risk to give noise trader portfolios an expected return edge whenever the noise trader share of agents is small. In the simulation the noise trader share never rises above 0.4, nor falls to zero.

Figure 8

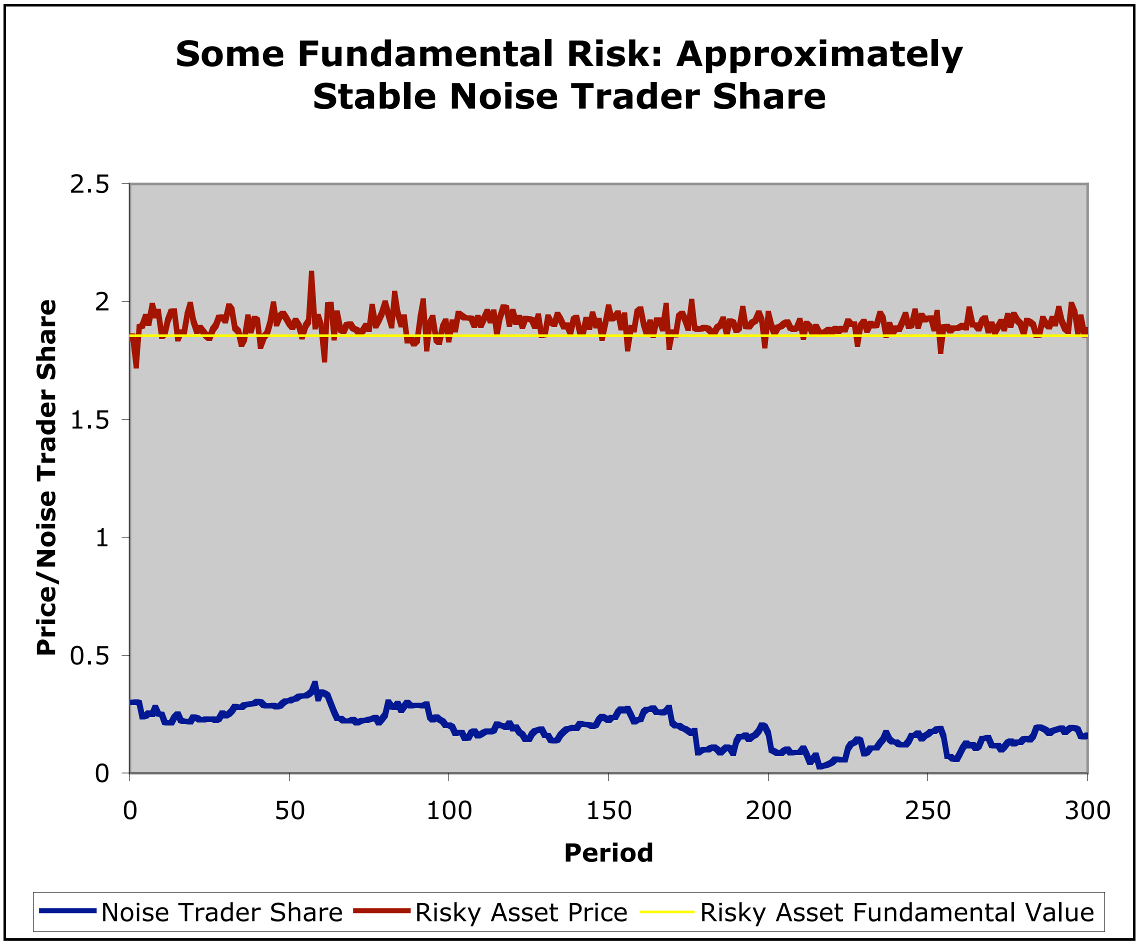
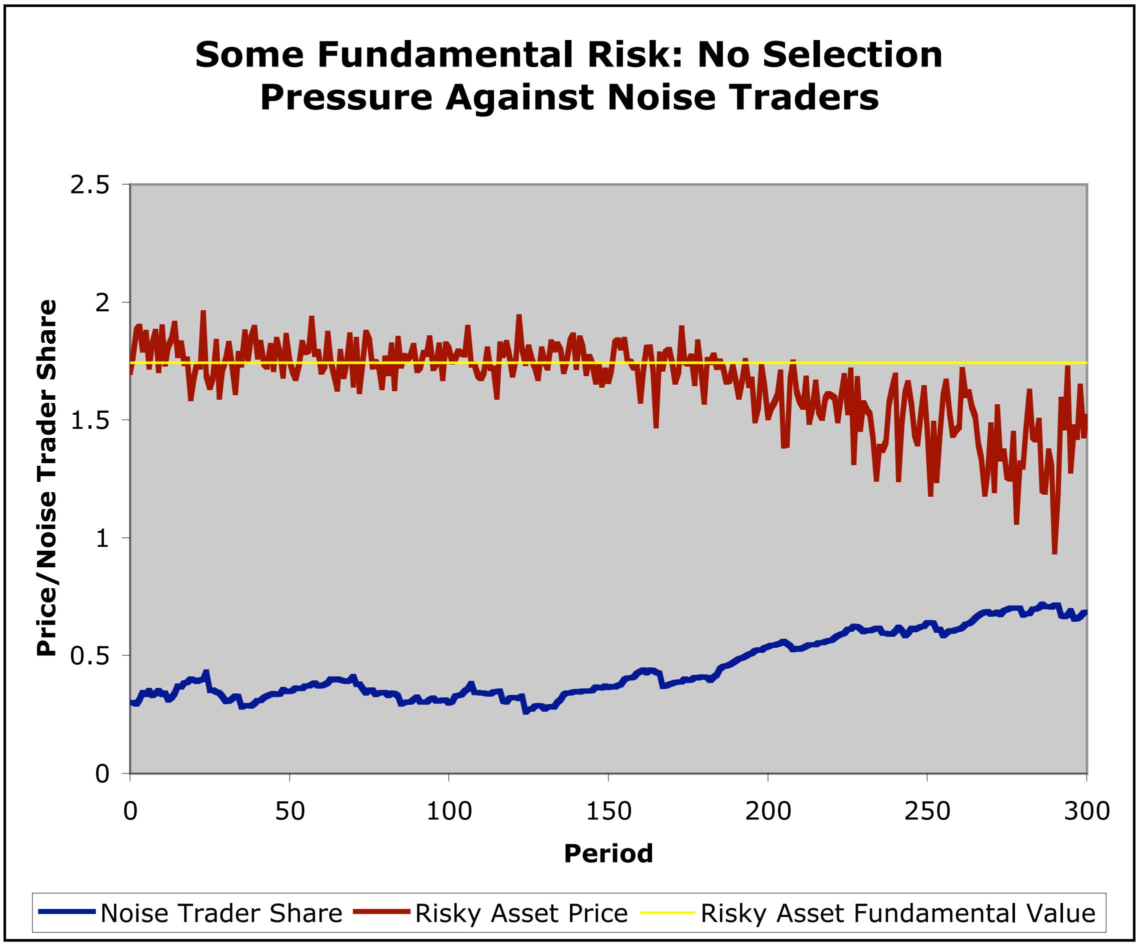


Figure 9 shows a simulation run in which there is enough fundamental risk to eliminate return-based selection pressure against noise traders. No matter what the noise trader share, the expected return on their portfolio is greater than the expected return on sophisticated investors' portfolio. In Figure 9, the share of noise traders climbs slowly and erratically but steadily: eventually it will reach close to 100%.

Figure 9



It is worth spending a moment looking at the time paths of prices in each of these simulation runs. They show a sample of the patterns of dynamic behavior that the model can generate. In the model, noise trader beliefs are i.i.d., hence the price series is very choppy—it has effectively no serial correlation. Contrast, in each simulation, the red realized price line with the narrow "fundamental price" line: the yellow line is what the price would be if all the agents in the economy were rational sophisticated investors. The short-term variability of the price series is, of course, proportional to the noise trader share: more noise traders mean more volatility of noise trader demand.

and in which over time sophisticated investors exploit noise traders, the noise trader share falls, and eventually noise traders are driven from the market. Thereafter noise traders do not reenter: since there is no fundamental risk in this simulation run, once the price of the risky asset is constant there is no noise trader portfolio to earn higher returns.

We have shown that risk created by the unpredictability of unsophisticated investors' opinions can significantly reduce the attractiveness of arbitrage. As long as arbitrageurs have short horizons and so must worry about liquidating their investment in a mispriced asset, their aggressiveness will be limited. Noise trading can thus lead to large divergences between market prices and fundamental values. And noise traders may be compensated for bearing risk, including the risk that they themselves create, and so earn higher returns than sophisticated investors even though they distort prices.

Given the traditional argument that the stock market price aggregates information and opinions, it is important to examine the extent to which there is a tendency of prices to reflect "good" rather than "bad" opinions. Even more than Figlewski's (1979) result that "bad" opinions can influence market prices for a long time, our paper suggests skepticism about the long run irrelevance of “bad” opinions.

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