The relative attractiveness of pay-as-you-go versus prefunded social-insurance systems depends to some degree on the gap between the return on capital \( r \) and the rate of real economic growth \( n+g \)—the sum of the rate of growth of employment \( n \) and the rate of growth of labor productivity \( g \). The larger is the rate of economic growth \( n+g \) relative to the return on capital \( r \), the more attractive do pay-as-you-go social-insurance systems become. When \( n+g \) approaches \( r \), they appear to be cheap and effective ways of increasing social welfare by passing resources down from the (rich and numerous) future to the (poor and relatively small) present.

The larger is \( r \) relative to \( n+g \), the greater are the benefits of prefunding social insurance systems. Prefunded systems can use high rates of return and compound interest to reduce the wedge between productivity and after-contribution real wages. They thus sacrifice the possibility of raising social welfare by moving wealth from the richer far future to the near future and the present, but in return they gain by reducing the social insurance tax rate and thus its deadweight loss.

To the extent that the political debate over the future of social insurance in America is conducted in the language of rational policy analysis, getting the gap between \( r \) on the one hand and \( n+g \) on the other
on the other hand right is important. Policies predicated on a false belief that \( r \) is much larger relative to \( n+g \) than it is will unduly burden the current and future young, and leave many disappointed when returns on assets turn out to be less than anticipated and prefunding leaves large unexpected holes in financing. Policies predicated on a false belief that \( n+g \) is higher relative to \( r \) than it in fact is pass up opportunities to lighten the overall tax burden and still provide near-equivalent income security benefits in the long run.

To see how this works, suppose that we have to raise an amount of revenue \( T \) for a social insurance program in the future, and we can either raise it now as a prefunded system by taxing the current generation which has per-capita income \( y \), or we can raise it in the future as a pay-as-you-go system by taxing the future generation. If we prefund it, it compounds over time: we only need to tax \( T/(1+r) \) now in order to have \( T \) available for social insurance in the future. If we go the pay-as-you-go route, the tax burden rests upon a larger and a richer generation: population grows at rate \( n \) and per-capita income grows at rate \( g \).

Let \( T_1 \) and \( T_2 \) be the per-capita taxes raised from the early and late generations, respectively. And let us take as our social welfare function the simple sum of the logs of consumption of the first and the second generation:

\[
SWF = \ln(Y - T_1) + (1 + n)\ln((1 + g)Y - T_2)
\]

where the \((1+n)\) arises because the later generation is larger than the earlier, where the \((1+g)\) enters because the later generation is richer than the earlier.

Because total taxes raised must add up in future value to the required revenue \( T \):
(2) \[ SWF = \ln(Y - T_1) + (1 + n)\ln\left((1 + g)Y - \frac{T - (1 + r)T_1}{(1 + n)}\right) \]

Looking for the maximum of the social welfare function:

(3) \[ 0 = -\frac{1}{Y - T_1} + \frac{(1 + r)}{(1 + g)Y - \frac{T - (1 + r)T_1}{(1 + n)}} \]

Which we can simply to:

(4) \[ (1 + r)(1 + n)Y - (1 + r)(1 + n)T_1 = (1 + g)(1 + n)Y - T + (1 + r)T_1 \]

And we arrive at:

(5) \[ T_1 = \frac{(r - g)(1 + n)Y}{(1 + r)(2 + n)} + \frac{T}{(1 + r)(2 + n)} \]

(6) \[ T_2 = \frac{(1 + n)T}{(2 + n)} + \frac{(g - r)(1 + n)Y}{(2 + n)} \]

We are done:

- If r=g, the optimal social planner raises a share \((1+n)/(2+n)\) of the total future value of taxes from the later generation—the faster is population growth, the lesser is the optimal degree of prefunding.
- If r goes up, she raises a lesser share from the later generation: the greater is the rate of return on assets, the greater is the optimal degree of prefunding.
- If g goes up, she raises a greater share from the later generation: the faster is the rate of productivity growth, the lesser is the optimal degree of prefunding.