Econ 101b - Answer Key to Problem Set 3

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Question 1

Suppose that the economy is well-described by the Solow growth model, with the diminishing-returns-to-capital parameter α = 1/3, the depreciation rate δ = .04, the population growth rate n=.02, and the rate of increase of the efficiency of labor g=.02.

Reminder: the Solow growth model is nothing else than the growth model presented throughout Chapter 4.

a. Suppose that the national savings rate s = 0.24, 24%. What is the steady-state capital output ratio?

$$\left(\frac{K}{Y}\right)_{ss} = \frac{s}{n + g + \delta} =$$

$$\frac{.24}{.02 + .02 + .04}$$

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3.
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b. Suppose that increased investment incentives and a large government budget surplus boost the

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$$\left(\frac{K}{Y}\right)_{ss} = \frac{s}{n + g + \delta} =$$

$$\frac{.32}{.02 + .02 + .04}$$

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4.
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c. Suppose that in the year 2000 the efficiency of labor E is $10,000 a year. What is the level of GDP per worker in 2000 if the economy is on the steady-state growth path given in (a)? What is the level of GDP per worker in 2030 if the economy remains on the steady-state growth path given in (a)?

$\left(\frac{Y}{L}\right)_t = \left(\frac{K}{Y}\right)_t^{\frac{\alpha}{1-\alpha}} E_t$. Thus in steady state,

$$\left(\frac{Y}{L}\right)_{2000,ss} = \left(\frac{K}{Y}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_{2000} = \left(\frac{s}{n+g+\delta}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_{2000} =$$

$$\left(\frac{.24}{.02 + .02 + .04}\right)^{\frac{1/3}{1-1/3}} 10000$$

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17320.5
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To know what the efficiency of labor is equal to in 2030, we will have to make use of differential equations. If you need help on this, refer to J.P.'s rough guide at:

http://www.well.com/user/jeanphi/100b/DiffEqu.doc

or to Lutfi's guide at:

http://econ161.berkeley.edu/stijns/DE.nb

What do we know about the efficiency of labor?

$\frac{\dot{E}_t}{E_t} = g \Leftrightarrow \dot{E}_t = g\,E_t \Leftrightarrow \partial_t\ E_t = g\,E_t$: this is our differential equation except that we have an initial condition here:

$\partial_t\ E_t = g\,E_t + 10000 = \partial_t\ E_t = .02\,E_t + 10000 \Leftrightarrow E_t = 10000\exp(.02\,t)$

```
Ef[t] = 10000 Exp[.02 t] /. t -> 30
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```
18221.2
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Thus, $\left(\frac{Y}{L}\right)_{2030,ss} = \left(\frac{K}{Y}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_{2030} = \left(\frac{s}{n+g+\delta}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_{2030} =$

$$\left(\frac{.24}{.02+.02+.04}\right)^{\frac{1/3}{1-1/3}}\ \%32$$

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31560.
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d. How much, in percentage terms, is the steady-state growth path given in (b) above the steady-state growth path given in (a)?

$$\frac{\left(\frac{Y}{L}\right)^{b}_{2030,ss}}{\left(\frac{Y}{L}\right)^{a}_{2030,ss}} = \frac{\left(\frac{s_b}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}}_{ss} E_t}{\left(\frac{s_a}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}}_{ss} E_t} = \frac{\left(\frac{.32}{.02+.02+.04}\right)^{\frac{1/3}{1-1/3}}}{\left(\frac{.24}{.02+.02+.04}\right)^{\frac{1/3}{1-1/3}}} =$$

$$\left(\frac{.32}{.24}\right)^{\frac{1/3}{1-1/3}}$$

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1.1547
```

i.e. output per worker will be approximatively 15% higher under scenario b than under scenario a *at any given point of time after the change in the savings rate, assuming the economy is on its steady state growth path.*

e. At approximately what rate will the capital output ratio converge towards its new steady state value in the year in which the savings rate is changed?

Recall from Chapter 4 that the rate at which the capital-output ratio converges to its steady-state is:

$$\left(\frac{\dot{K/Y}}{K/Y}\right)_t = (1-\alpha)(n+g+\delta)\left(\frac{(K/Y)_{ss}-(K/Y)_t}{(K/Y)_t}\right) =$$

$$(1-1/3)\ (.02+.02+.04)\ \left(\frac{4-3}{3}\right)$$

```
0.0177778
```

Thus the capital-output ratio converges to its steady-state at a rate of approximately 1.8% of the proportional difference between the steady-state capital-output ratio and the current (here previous steady-state) capital-output ratio.

f. At approximately what rate will output per capita grow in the year in which the savings rate is changed?

$$\left(\frac{Y}{L}\right)_t = \left(\frac{K}{Y}\right)_t^{\frac{\alpha}{1-\alpha}} E_t \Rightarrow \left(\frac{\dot{Y/L}}{Y/L}\right)_t =$$

$$\frac{\alpha}{1-\alpha}\left(\frac{\dot{K/Y}}{K/Y}\right)_t + \frac{\dot{E}_t}{E_t} \Rightarrow \left(\frac{\dot{Y/L}}{Y/L}\right)_t - g = \frac{\alpha}{1-\alpha}(1-\alpha)(n+g+\delta)\left(\frac{(K/Y)_{ss}-(K/Y)_t}{(K/Y)_t}\right) \Rightarrow$$

$$\left(\frac{\dot{Y/L}}{Y/L}\right)_t - g = \alpha(n+g+\delta)\left(\frac{(K/Y)_{ss}-(K/Y)_t}{(K/Y)_t}\right) =$$

```
1/3 (.02 + .02 + .04) ((4 - 3)/3)
```

```
0.00888889
```

Thus output per worker converges to its steady-state growth path at a rate of approximately 0.9% of the proportional difference between the steady-state capital-output ratio and the current (here previous steady-state) capital-output ratio.

💡 Notice that output per worker converges to its steady-state growth path at half the rate of at which the capital-output ratio converges to its steady-state.

Question 2

Suppose that the economy is well-described by the Solow growth model, with the diminishing-returns-to-capital parameter α = 1/4, the depreciation rate δ = .04, the population growth rate n=.02, and the rate of increase of the efficiency of labor g=.02.

💡 Reminder: the Solow growth model is nothing else than the growth model presented throughout Chapter 4.

a. Suppose that the national savings rate s = 0.24, 24%. What is the steady-state capital-output ratio? If the efficiency of labor in year 2000 is equal to $20,000 a year, what is the steady-state growth path level of output per worker in 2000? What is the steady-state growth path level of consumption per worker in 2000?

$$\left(\frac{K}{Y}\right)_{ss} = \frac{s}{n+g+\delta} =$$

```
     .24
-------------
.02 + .02 + .04
```

```
3.
```

$\left(\frac{Y}{L}\right)_t = \left(\frac{K}{Y}\right)_t^{\frac{\alpha}{1-\alpha}} E_t$. Thus in steady state,

$$\left(\frac{Y}{L}\right)_{2000,ss} = \left(\frac{K}{Y}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_{2000} = \left(\frac{s}{n+g+\delta}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_{2000} =$$

$$\left(\frac{.24}{.02+.02+.04}\right)^{\frac{1/4}{1-1/4}} 20000$$

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28845.
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$$\left(\frac{C}{L}\right)_t = (1-s)\left(\frac{Y}{L}\right)_t \Rightarrow \left(\frac{C}{L}\right)_{2000,ss} = (1-s)\left(\frac{Y}{L}\right)_{2000,ss} = (1-s)\left(\frac{K}{Y}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_{2000} = (1-s)\left(\frac{s}{n+g+\delta}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_{2000} =$$

$$(1-.24)\left(\frac{.24}{.02+.02+.04}\right)^{\frac{1/4}{1-1/4}} 20000$$

```
21922.2
```

b. Suppose that the national savings rate s = 0.32, 32%. What is the steady-state capital-output ratio? If the efficiency of labor in year 2000 is equal to $20,000 a year, what is the steady-state growth path level of output per worker in 2000? What is the steady-state growth path level of consumption per worker in 2000?

$$\left(\frac{K}{Y}\right)_{ss} = \frac{s}{n+g+\delta} =$$

$$\frac{.32}{.02 + .02 + .04}$$

4.

$\left(\frac{Y}{L}\right)_t = \left(\frac{K}{Y}\right)_t^{\frac{\alpha}{1-\alpha}} E_t$. Thus in steady state,

$$\left(\frac{Y}{L}\right)_{2000,ss} = \left(\frac{K}{Y}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_{2000} = \left(\frac{s}{n+g+\delta}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_{2000} =$$

$$\left(\frac{.32}{.02 + .02 + .04}\right)^{\frac{1/4}{1-1/4}} 20000$$

31748.

$$\left(\frac{C}{L}\right)_t = (1-s)\left(\frac{Y}{L}\right)_t \Rightarrow \left(\frac{C}{L}\right)_{2000,ss} = (1-s)\left(\frac{Y}{L}\right)_{2000,ss} = (1-s)\left(\frac{K}{Y}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_{2000} = (1-s)\left(\frac{s}{n+g+\delta}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_{2000} =$$

$$(1 - .32)\left(\frac{.32}{.02 + .02 + .04}\right)^{\frac{1/4}{1-1/4}} 20000$$

21588.7

c. Can you explain why your level of consumption per worker in case (b) was different from your level of consumption per worker in case (a)? Was it lower or higher

As we can see consumption per worker is lower in case (b) than in case (a) even though the higher savings rate has secured higher output per worker. Recall that the Golden Rule savings rate is s=α. Since α=1/4=.25, going from (a) to (b), the savings was raised way above the Golden Rule savings rate. Intuitively, increasing the savings always raises output per worker since the marginal productivity of capital is always positive. However, the marginal productivity of capital is decreasing, thus above the Golden Rule level of output per worker, the increase in output per worker does not make up anymore for the associated increase in capital depreciation per unit of effective labor. Does consumption per worker, i.e. output per worker minus gross investment per worker actually goes down.

Question 3

Suppose that population growth depends on the level of output per worker, so that:

n = (.0001) x [(Y/L) - $200]

the population growth rate n is zero if output per worker equals $200, and that each $100 increase in output per worker raises the population growth rate by 1% per year. Suppose also that the economy is in its Malthusian regime, so that the rate of increase of the efficiency of labor E is zero and output per worker is given by:

$$\left(\frac{Y}{L}\right)_t = \left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}} E_0$$

with the diminishing-returns-to-investment parameter α = .5, with the depreciation rate δ = .04, and with the efficiency of labor E_0 = $100.

a. Suppose that the savings rate s is equal to .08, 8% per year. Graph (on the same set of axes) steady-state output-per-worker (Y/L) as a function of the population growth rate n from equation (2) and the population growth rate n as a function of output-per-worker (Y/L) from equation (1).

Let me denote output per worker as y, and n as n[y], then (1) can be rewritten as:

```
n[y] == 0.0001 (-200 + y)
```

Once we plug parameter values into (2), it can written as:

$$y = \left(\frac{0.08}{n + .04}\right) 100 \Leftrightarrow n = \frac{8}{y} - .04$$

Denoting again output per worker as y, and n as n[y], then (1) can be rewritten as

$$n[y] == \frac{8}{y} - .04$$

Let's plot these two functions with output per worker on the x-axis and the rate of population growth on the y-axis:

```
Plot[{8/y - .04, .0001 (y - 200)}, {y, 0, 1000},
 PlotStyle -> {{RGBColor[1, 0, 0]}, {RGBColor[0, 1, 0]}},
 AxesLabel -> {"y", "n"}]
```

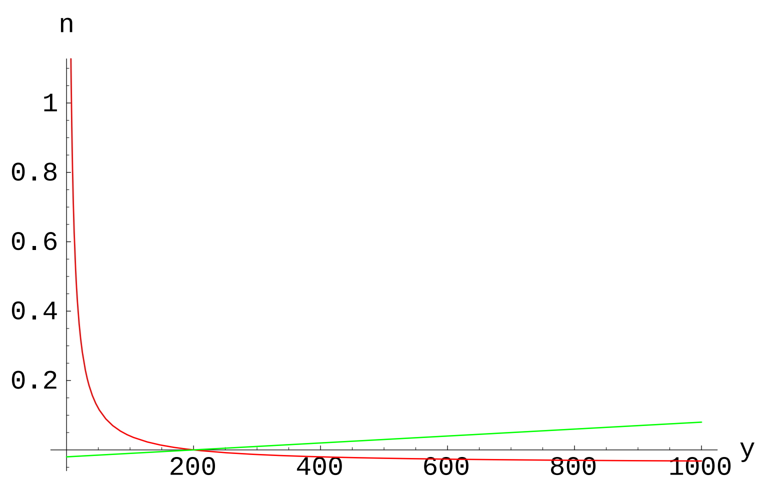

Figure 1

b. Where do the curves cross? For what levels of output per worker Y/L and population growth n is the economy (i) on its steady-state path, and (ii) at its Malthusian rate of population growth?

The (acceptable) solution to this system is (as can be seen just reading the graph above):

```
Solve[{n == 8/y - .04, n == .0001 (y - 200)}, {n, y}]
```

```
{n → 0., y → 200.}
```

Thus, equilibrium in this model requires output per worker of $200 and a constant population. Notice that this is not so bizzare as below $200, population growth is positive and above $200, negative, implying that our solution is stable.

c. Suppose that the savings rate were to rise by an infinitesimal amount--say by one-hundredth of one percentage point, from .08 to .0801. Calculate approximately how the equilibrium position of the economy would change. By how much --and in which direction-- would steady-state outputper worker change? By how much --and in which direction-- would the population growth rate change?

Let's go back to our system of equation keeping this s open to take any value:

$$n == -0.04 + \frac{100\ s}{y} \tag{1}$$

n==0.0001 (-200+y) (2)

The solution to (3) and (4) is:

$$\left\{n \to 0.0001 \left(-300. + 0.5 \sqrt{40000. + 4. \times 10^6\ s}\right),\ y \to 0.5 \left(-200. + \sqrt{40000. + 4. \times 10^6\ s}\right)\right\}$$

The derivative with respect to s of the solution for n is:

$$\frac{100.}{\sqrt{40000. + 4. \times 10^6\ s}}$$

The resulting change in n^* is going can be approximated by $\partial_s\ n^*\ \Delta s =$

0.0000166667

Thus with $\Delta s = .0001$, the resulting change in n^* is going to be approximatly 2 thousands of a percent.

The derivative with respect to s of the solution for y is:

$$\frac{1. \times 10^6}{\sqrt{40000. + 4. \times 10^6\ s}}$$

The resulting change in y^* can be approximated by $\partial_s\ n^*\ \Delta s =$

0.166667

Thus with $\Delta s = .0001$, the resulting change in y^* is going to be approximatly $0.16.

- One can perhaps answer this question more elegantly using the Implicit Function Theorem, but J.P. is getting tired... And as we say in French, "la fin justifie les moyens" or more colorfully "qu'importe la bouteille pourvu que vienne l'ivresse".

Question 4.

Suppose that the economy is well described by the Solow growth model, with the diminishing-returns-to-investment parameter α= 1/2, the depreciation rate δ = .03, the population growth rate n=.01, and the rate of increase of the efficiency of labor g=.01. Suppose that the savings rate s = 20 and that in year 2000 the efficiency of labor E is $10,000.

a. What is the steady-state capital-output ratio?

$$\left(\frac{K}{Y}\right)_{ss} = \frac{s}{n+g+\delta} =$$

```
      .20
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.01 + .01 + .03
```

```
4.
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b. What is the steady-state level of output per worker Y/L in 2000?

$\left(\frac{Y}{L}\right)_t = \left(\frac{K}{Y}\right)_t^{\frac{\alpha}{1-\alpha}} E_t$. Thus in steady state,

$$\left(\frac{Y}{L}\right)_{2000,ss} = \left(\frac{K}{Y}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_{2000} = \left(\frac{s}{n+g+\delta}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_{2000} =$$

$$\left(\frac{.20}{.01+.01+.03}\right)^{\frac{1/2}{1-1/2}} 10000$$

```
40000.
```

c. Suppose that actual output per worker in 2000 is $35,000. Is output per worker above or below its steady-state value?

Below.

d. Using the approximation that each year the economy closes a fraction:
(1-α) x (n + g + delta)
of the gap between its current level of output per worker and its steady-state value of output per worker, calculate (approximately) what output per worker will be in 2001.

(1-α) (n + g + δ)=

$$\left(1-\frac{1}{2}\right)(.01+.01+.03)$$

```
0.025
```

This economy closes 2.5% of the gap in output per worker with its steady state growth path.

$$(1-\alpha)(n+g+\delta)\left[\left(\frac{Y}{L}\right)_{2000,ss}-\left(\frac{Y}{L}\right)_{2000}\right]=$$

```
.025 * 5000
```

```
125.
```

This corresponds to an increase in output per worker of the order 125. Taking account of the usual increase in labot efficiency (g) and of this closing of the gap:

$$\left(\frac{Y}{L}\right)_{2000}=$$

```
35000 (1.01) + 125
```

```
35475.
```

e. Using the same approximation, what (approximately) will output per worker be in 2010?

A very rough way of approximating output per worker in 2010 is:

$\left(\frac{Y}{L}\right)_{2010} =$

```
35000 (1.01)^10 + 125 * 10
```

```
39911.8
```

f. Do you think the above approximation formula works well when we project far into the future? Why or why not? (Phrase your answer in terms of economics.)

It's unlikely to be a good approximation if we project far into the future since the change in output due to convergence is going to be decreasing as income per worker closes its gap with its steady state growth path. The farther an economy is from steady-state the stronger the convergence forces at work.

Question 5

Suppose somebody who hasn't taken any economics courses were to ask you why humanity escaped from the Malthusian trap--of very low standards of living and slow population growth rates that nevertheless put pressure on available natural resources and kept output per worker from rising--in which humanity found itself between the year 8000 B.C.E. and 1800. What answer would you give? (One paragraph only, please!)

Essentially, technological progress won the battle with Mathusian dynamics. Given the fact that the population growth rate becomes a decreasing function of output per worker above the level of output per worker which defines the demographic transition. Even though most of the increases in productivity had been absorbed by increases in the population, once the critical level of output per worker was to be attained, humanity escaped the Malsthusian trap.

Question 6

Suppose somebody who hasn't taken any economics courses were to ask you why it is that
some countries are so very, very much poorer than others in the world today. What answer would you give? (One paragraph only, please!)

- One view one can have on this is that differences in saving / investment rates (both terms in physical and human capital) and population growth rates explain about 80% of the differences in output per worker accross countries.

- Another view people have is that differences in "Social Infrastruce" is the ultimate determinant of all other relevant (but proximate) variables determining a country's economic fate; we're then back to some kind of cultural explanation of productivity differences *a la* Veblen.

- Third, one can take the pragmatic view that a large chunk of differences in output per worker has been due to differences in economic policy. For example, being or having been communist systematically leaves a country with a lower level of productivity, *ceteris paribus*.