Econ 101b - Answer Key to Problem Set 4

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Question 1

The original capital-output ratio was:

$$\left(\frac{K}{Y}\right)_{ss,old} = \frac{s}{n+g+\delta} =$$

```
.16/(.01 + .01 + .03)
```

```
3.2
```

Using our knowledge of first-order differential equations:

$$E_{2000} = 30000 \Longrightarrow E_{2040} =$$

```
30000 Exp[40 * .01]
```

```
44754.7
```

Assuming the economy was originally in steady-state output per worker in 2040 will be:

$$\left(\frac{Y}{L}\right)_{ss,old,2040} = \left(\frac{K}{Y}\right)_{ss,old}^{\frac{\alpha}{1-\alpha}} E_{2040} =$$

```
80059.7
```

The new steady-state capital-output ratio is:

$$\left(\frac{K}{Y}\right)_{ss,new} = \frac{s}{n+g+\delta} =$$

```
.20 / (.01 + .01 + .03)
```

```
4.
```

Out of steady state,the rate of groth of the capital-outputratio is:

$$\left(\frac{\dot{K/Y}}{K/Y}\right)_t = (1-\alpha)(n+g+\delta)\left(\frac{(K/Y)_{ss}-(K/Y)_t}{(K/Y)_t}\right) \Leftrightarrow$$

$$\left(\frac{\dot{Z}}{Z}\right)_t = (1-\alpha)(n+g+\delta)\left(\frac{Z_{ss}-Z_t}{Z_t}\right) \Leftrightarrow$$

$$\dot{Z}_t = (1-\alpha)(n+g+\delta)(Z_{ss}-Z_t) \Leftrightarrow$$

$$\dot{Z}_t = -\tau Z_t + \tau Z_{ss} \Leftrightarrow$$

From our study of differential equations, we know that:

$$\dot{X}_t = X_t + b \Leftrightarrow X_t = \left(X_0 + \frac{b}{a}\right)e^{at} - \frac{b}{a} \Leftrightarrow$$

Applying this to our differential equation for the capital-outputratio:

$$Z_t = \left(Z_0 + \frac{\tau Z_{ss}}{-\tau}\right)e^{-\tau t} - \frac{\tau Z_{ss}}{-\tau} \Leftrightarrow$$

$$Z_t = (Z_0 - Z_{ss})e^{-\tau t} + Z_{ss} = Z_0 e^{-\tau t} + Z_{ss}(1-e^{-\tau t})$$

The rate of convergenceis:

τ=(1-α)(n+g+δ)=

```
(1 - 1 / 3) (.01 + .01 + .03)
```

```
0.0333333
```

We can now find the capital-outputratio in 2040:

$$Z_t = 3.2\,e^{-.0333333t} + 4\,(1-e^{-.033333t}) \Leftrightarrow \left(\frac{K}{Y}\right)_{2040} = Z_{2040} =$$

```
3.78912
```

Now we can say what output per worker will be in 2040:

$$\left(\frac{Y}{L}\right)_{ss,new,2040} = \left(\frac{K}{Y}\right)_{ss,new}^{\frac{\alpha}{1-\alpha}} E_{2040} =$$

```
87118.1
```

Notice that 40 years ahead this reduction in the deficit has brought about a 9% increase in material standards of living. Alan Blinder was thus right in warning the Clinton team that budgetary efforts only lead to substancial increases in output per worker over the long-run...

Question 2

$E_{2000} = 15000 \Longrightarrow E_{2040} =$

```
15000 Exp[40 * .01]
```

```
22377.4
```

$\left(\frac{Y}{L}\right)_{ss,old,2040} = \left(\frac{K}{Y}\right)_{ss,old}^{\frac{\alpha}{1-\alpha}} E_{2040} =$

```
71607.6
```

τ=(1-α)(n+g+δ)=

```
(1 - 1 / 2) (.01 + .01 + .03)
```

```
0.025
```

$Z_t = 3.2\, e^{-.025t} + 4\,(1 - e^{-.025t}) \Leftrightarrow \left(\frac{K}{Y}\right)_{2040} = Z_{2040} =$

```
3.7057
```

$\left(\frac{Y}{L}\right)_{ss,new,2040} = \left(\frac{K}{Y}\right)_{ss,new}^{\frac{\alpha}{1-\alpha}} E_{2040} =$

```
82923.7
```

Notice that 40 years ahead this reduction in the deficit has brought about a 16% increase in material standards of living. Thus, when the curbature of the production function is less pronounced, i.e. when the dimishing marginal returns to capital kick in more slowly, the increase in output per worker is more important. The change in initial efficiency does not matter for this purpose.

Question 3

The original capital-outputratio was:

$$\left(\frac{K}{Y}\right)_{ss,old} = \frac{s}{n+g+\delta} =$$

```
     .20
.01 + .01 + .03
```

```
4.
```

Using our knowledge of first-order differential equations:

$$E_{2000} = 30000 \Longrightarrow E_{2040} =$$

```
15000 Exp[40 * .01]
```

```
22377.4
```

Assuming the economy was originally in steady-state output per worker in 2040 will be:

$$\left(\frac{Y}{L}\right)_{ss,old,2040} = \left(\frac{K}{Y}\right)_{ss,old}^{\frac{\alpha}{1-\alpha}} E_{2040} =$$

```
44754.7
```

The new steady-state capital-outputratio is:

$$\left(\frac{K}{Y}\right)_{ss,new} = \frac{s}{n+g+\delta} =$$

```
     .20
.01 + .02 + .03
```

```
3.33333
```

Out of steady state, the rate of groth of the capital-outputratio is:

$$\left(\frac{\dot{K/Y}}{K/Y}\right)_t = (1-\alpha)(n+g+\delta)\left(\frac{(K/Y)_{ss}-(K/Y)_t}{(K/Y)_t}\right) \Leftrightarrow$$

From our study of differential equations, we know that:

$$Z_t = (Z_0 - Z_{ss})\, e^{-\tau t} + Z_{ss} = Z_0\, e^{-\tau t} + Z_{ss}\,(1 - e^{-\tau t})$$

The rate of convergence is:

τ=(1-α)(n+g+δ)=

```
(1 - 1 / 3) (.01 + .02 + .03)
```

```
0.04
```

We can now find the capital-output ratio in 2040:

$$Z_t = 4\, e^{-.04\, t} + 3.3\, (1 - e^{-.04\, t}) \Leftrightarrow \left(\frac{K}{Y}\right)_{2040} = Z_{2040} =$$

```
3.46793
```

Using our knowledge of first-order differential equations:

$$E_{2000} = 30000 \Longrightarrow E_{2040} =$$

```
15000 Exp[40 * .02]
```

```
33383.1
```

Now we can say what output per worker will be in 2040:

$$\left(\frac{Y}{L}\right)_{ss,new,2040} = \left(\frac{K}{Y}\right)_{ss,new}^{\frac{\alpha}{1-\alpha}} E_{2040} =$$

```
62167.3
```

Notice that 40 years ahead this reduction in the deficit has brought about a 39% increase in material standards of living. Even though the higher labor efficiency growth rate results in a new lower steady-state capital-ouput ratio, the higher growth rate will actually bring about a higher output per worker; 40 years is long enough to let the growth effect dominate the level effect.

Question 4

Using our knowledge of first-order differential equations:

$E_{2000} = 15000 \Longrightarrow E_{2040} =$

```
30000 Exp[40 * .01]
```

```
44754.7
```

Assuming the economy was originally in steady-state output per worker in 2040 will be:

$\left(\frac{Y}{L}\right)_{ss,old,2040} = \left(\frac{K}{Y}\right)_{ss,old}^{\frac{\alpha}{1-\alpha}} E_{2040} =$

```
179019.
```

The rate of convergence is:

τ=(1-α)(n+g+δ)=

```
(1 - 1 / 2) (.01 + .02 + .03)
```

```
0.03
```

We can now find the capital-output ratio in 2040:

$Z_t = 4\, e^{-.03\,t} + 3.3\,(1 - e^{-.03\,t}) \Leftrightarrow \left(\frac{K}{Y}\right)_{2040} = Z_{2040} =$

```
3.53413
```

Using our knowledge of first-order differential equations:

$E_{2000} = 30000 \Longrightarrow E_{2040} =$

```
30000 Exp[40 * .02]
```

```
66766.2
```

Now we can say what output per worker will be in 2040:

$\left(\frac{Y}{L}\right)_{ss,new,2040} = \left(\frac{K}{Y}\right)_{ss,new}^{\frac{\alpha}{1-\alpha}} E_{2040} =$

```
235960.
```

Notice that 40 years ahead this reduction in the deficit has brought about a 32% increase in material standards of living. The lower marginal return to investment parameter implies a slower rate of convergence; over the course of 40 years this effect has decreased the improvement in living standards resulting from the increase in the rate of growth.

Question 5

The original capital-output ratio was:

$$\left(\frac{K}{Y}\right)_{ss,old} = \frac{s}{n+g+\delta} =$$

```
.16 / (.025 + .025 + .03)
```

```
2.
```

$$\left(\frac{Y}{L}\right)_{ss,2000} = 10000 = \left(\frac{K}{Y}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_{2000} = 2^{\frac{1/2}{1-1/2}} E_{2000} \Leftrightarrow E_{2000} =$$

```
10000 / 2
```

```
5000
```

Using our knowledge of first-order differential equations:

$$E_{2000} = 5000 \Longrightarrow E_{2040} =$$

```
5000 Exp[40 * .025]
```

```
13591.4
```

Assuming the economy was originally in steady-state output per worker in 2040 will be:

$$\left(\frac{Y}{L}\right)_{ss,2040} = \left(\frac{K}{Y}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_{2040} = 2^{\frac{1/2}{1-1/2}} E_{2040} =$$

```
27182.8
```

Question 6

A.

The new capital-outputratio is:

$$\left(\frac{K}{Y}\right)_{ss,new} = \frac{s}{n+g+\delta} =$$

```
     .20
.025 + .025 + .03
```

```
2.5
```

Out of steady state,the rate of groth of the capital-outputratio is:

$$\left(\frac{\dot{K/Y}}{K/Y}\right)_t = (1-\alpha)(n+g+\delta)\left(\frac{(K/Y)_{ss} - (K/Y)_t}{(K/Y)_t}\right) \Leftrightarrow$$

From our study of differential equations, we know that:

$$Z_t = (Z_0 - Z_{ss})\, e^{-\tau t} + Z_{ss} = Z_0\, e^{-\tau t} + Z_{ss}\, (1 - e^{-\tau t})$$

The rate of convergenceis:

τ=(1-α)(n+g+δ)=

```
(1 - 1 / 2) (.025 + .025 + .03)
```

```
0.04
```

We can now find the capital-outputratio in 2040:

$$Z_t = 2\, e^{-.04 t} + 2.5\,(1 - e^{-.04 t}) \Leftrightarrow \left(\frac{K}{Y}\right)_{2040} = Z_{2040} =$$

```
2.39905
```

Now we can say what output per worker will be in 2040:

$$\left(\frac{Y}{L}\right)_{ss,new,2040} = \left(\frac{K}{Y}\right)_{ss,new}^{\frac{\alpha}{1-\alpha}} E_{2040} =$$

```
32606.5
```

B.

The new capital-outputratio is:

$$\left(\frac{K}{Y}\right)_{ss,new} = \frac{s}{n+g+\delta} =$$

```
.16 / (.01 + .025 + .03)
```

```
2.46154
```

Out of steady state,the rate of groth of the capital-outputratio is:

$$\left(\frac{\dot{K/Y}}{K/Y}\right)_t = (1-\alpha)(n+g+\delta)\left(\frac{(K/Y)_{ss} - (K/Y)_t}{(K/Y)_t}\right) \Leftrightarrow$$

From our study of differentialequations, we know that:

$$Z_t = (Z_0 - Z_{ss})\, e^{-\tau t} + Z_{ss} = Z_0\, e^{-\tau t} + Z_{ss}\,(1 - e^{-\tau t})$$

The rate of convergenceis:

τ=(1-α)(n+g+δ)=

```
(1 - 1 / 2) (.01 + .025 + .03)
```

```
0.0325
```

We can now find the capital-outputratio in 2040:

$$Z_t = 2\, e^{-.0325t} + 2.46\,(1 - e^{-.0325t}) \Leftrightarrow \left(\frac{K}{Y}\right)_{2040} = Z_{2040} =$$

```
2.33575
```

Now we can say what output per worker will be in 2040:

$$\left(\frac{Y}{L}\right)_{ss,new,2040} = \left(\frac{K}{Y}\right)_{ss,new}^{\frac{\alpha}{1-\alpha}} E_{2040} =$$

```
31746.2
```

C.

The new capital-outputratio is:

$$\left(\frac{K}{Y}\right)_{ss,new} = \frac{s}{n+g+\delta} =$$

$$\frac{.20}{.01 + .025 + .03}$$

```
3.07692
```

Out of steady state,the rate of groth of the capital-outputratio is:

$$\left(\frac{\dot{K/Y}}{K/Y}\right)_t = (1-\alpha)(n+g+\delta)\left(\frac{(K/Y)_{ss} - (K/Y)_t}{(K/Y)_t}\right) \Leftrightarrow$$

From our study of differential equations, we know that:

$$Z_t = (Z_0 - Z_{ss})\, e^{-\tau t} + Z_{ss} = Z_0\, e^{-\tau t} + Z_{ss}\,(1 - e^{-\tau t})$$

The rate of convergenceis:

τ=(1-α)(n+g+δ)=

```
(1 - 1 / 2) (.01 + .025 + .03)
```

```
0.0325
```

We can now find the capital-outputratio in 2040:

$$Z_t = 2\, e^{-.0325 t} + 3.1\,(1 - e^{-.0325 t}) \Leftrightarrow \left(\frac{K}{Y}\right)_{2040} = Z_{2040} =$$

```
2.78343
```

Now we can say what output per worker will be in 2040:

$$\left(\frac{Y}{L}\right)_{ss,new,2040} = \left(\frac{K}{Y}\right)_{ss,new}^{\frac{\alpha}{1-\alpha}} E_{2040} =$$

```
37830.7
```

- The change in the savings rate bought Mexico a 20% increase the GDP per worker over the course of 40 years. The demographic change bought Mexico a 17% increase the GDP per worker over the course of 40 years. The combination of both of these changes bought Mexico a 39% increase the GDP per worker over the course of 40 years which is more than the sum of the effect of both the increase in savings and the demographic change. Recall that convergence is faster at first so a larger change in the relative difference in capital-output ratio gives rise to a faster rate of convergence.

Question 7

The economics profession seems devided to me on this question. On the one hand there are powerful convergence force at work. Also, we may think that, at last, developping countries may do their demographic transition. On the other hand, very poor social infrastructure and bad economic policies have tended to prevend convergence to happen at the rate that our simple growth model would have us expect. One of the keys to this puzzle seem to be education, and especially, elementary education for women (because increased women education fastens the demographic transition.) International development and financial aid is also said to be very often ill-suited and to provide wrong incentives to governments of developping countries.