

Econ 101b - Answer Key to Problem Set 5

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Note: This answer key is partly based on a notebook written by Lutfi Latif.

Question 1

Suppose that the consumption, investment, net exports, and exchange rate functions are

$$\text{Cons}(C0, c, t, Y) := C0 + \alpha(1-t) Y$$

$$\text{Inv}(I0, \alpha, r) := I0 - \alpha r$$

$$\text{IM}(\mu, Y) := \mu Y$$

$$\epsilon(\epsilon0, \phi, r, rf) := \epsilon0 + (r-rf)\phi$$

$$\text{GX}(x, Yf, \chi, rf) := x Yf + \chi \epsilon(100, 10, r, rf)$$

A. Derive the IS curve for this economy: real GDP as a function of all the unspecified variables in the economy.

Equate Income to Planned Expenditure, leaving unspecified the unknown variables:

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Solve [Y == Cons [3000, 0.5, 0.4, Y] +
      Inv [1200, 100, r] + G + GX [0.1, Yf, 4, rf] - IM [0.2, Y], Y] [[1,
      111
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And isolate national income on the LHS as a function of the rest of the variables:

$$Y \rightarrow -1.11111(-1.G + 100.r - 4.(10.(r - 1.rf) + 100) - 0.1 Yf - 4200)$$

B. Suppose that the foreign interest rate r_f is 5%, that total foreign income Y_f is \$10000, and that government spending G is \$3000.

Just substitute these values in the previous expression:

$$Y \rightarrow -1.11111 (-8200. - 4. (100. + 10. (-5. + r)) + 100. r)$$

... and simplify:

$$Y \rightarrow 9333.33 - 66.6667 r$$

C. What then is equilibrium annual real GDP if the central bank sets the real interest rate at 3%? At 5%? At 7%?

We have National Income as a function of the interest rate:

$$9333.33 - 66.6667 r$$

Let's just plug in for the various interest rates:

3 percent:

$$9133.33$$

5 percent:

$$9000.$$

7percent:

$$8866.67$$

We can invert this expression and get the interest rate as a function of the equilibrium level of income:

$$140. - 0.015 Y$$

And of course we can draw this function, especially since we know it's a line and we have (more than) 2 points lying on it:

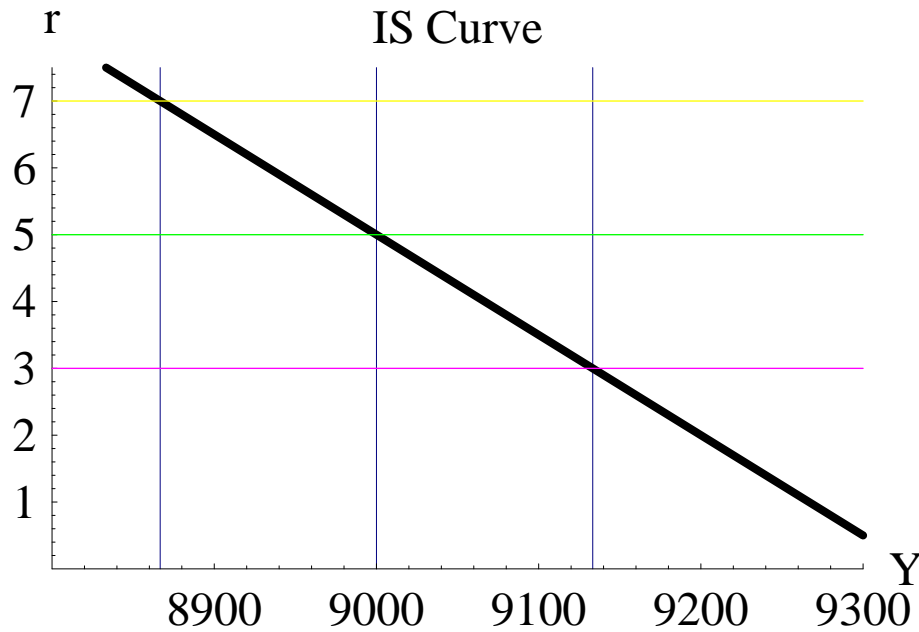


Figure 1

Question 2

Suppose further that the government follows a balanced budget rule: government purchases G are equal to government tax collections tY .

Derive the IS for this economy: real GDP as a function of all the unspecified variables in the economy.

$$\text{Cons}[C0, c, t, Y_] := C0 + c(1-t)Y$$

$$\text{Inv}[I0, \alpha, r_] := I0 - \alpha r$$

$$\text{IM}[\mu, Y_] := \mu Y$$

$$\epsilon[\epsilon0, \phi, r, rf_] := \epsilon0 + \phi(r - rf)$$

$$\text{GX}[x, Yf, \chi, rf_] := x Yf + \chi \epsilon[100, 10, r, rf]$$

$$G[t, Y_] := (1-t)Y$$

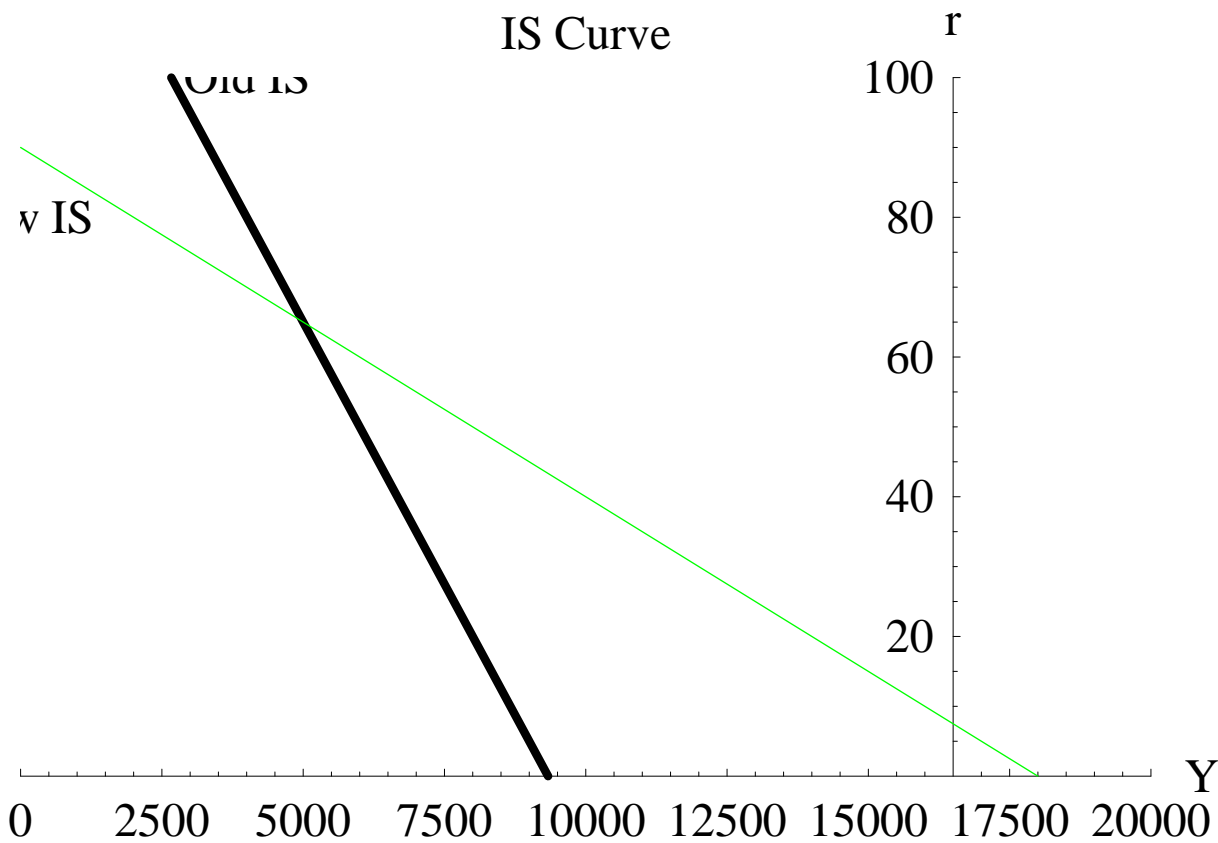
Equilibrium income as a function of the interest rate is:

$$18666.7 - 200 \cdot r - 133.333 \cdot r^f$$

This is the IS Curve.

Is the level of real GDP along the IS curve more or less sensitive to changes in interest rates than in problem 1? Why?

The level of real GDP along the IS curve is more sensitive to changes in interest rates than in problem 1 because now anytime income increases, taxes and thus government spending also increase, providing a further boost to income. Conversely, the coefficient on r is now -200 instead of -60 , so anytime the interest rate is increased, income decreases relatively more when there is a balanced budget spending.



Question 3

Suppose that the economy is short of its full-employment level of GDP, \$8 trillion, by \$500 billion, with the MPC out of disposable income equal to 0.6, the import share μ equal to 0.2, and the tax rate t equal to 25%.

- a. Suppose the government wants to boost real GDP up to full employment by cutting taxes. How large a cut in the tax rate is required to boost real GDP to full employment? How large a cut in total tax collections is produced by this cut in the tax rate?

$$\mu=0.2; c=0.6; t=0.25; Y=7500; \Delta Y=500;$$

First, we need to solve for the (unknown) level of autonomous spending.

$$\text{NSolve}\left[Y == \frac{A}{1 - c + c(t) + \mu}, A\right]$$

$$\{\{A \rightarrow 5625.\}\}$$

The intercept of the planned expenditure line, A , is 5625 and the slope is :

$$m_0 = \frac{7500 - 5625}{7500 - 0}$$

$$0.25$$

This is due to the fact that equilibrium income is equal to 7500, thus the aggregate expenditure line has to go through (7500, 7500). Which is what we can also get from looking at the expenditure function and grouping together terms involving national income:

$$c(1 - t) - \mu$$

$$0.25$$

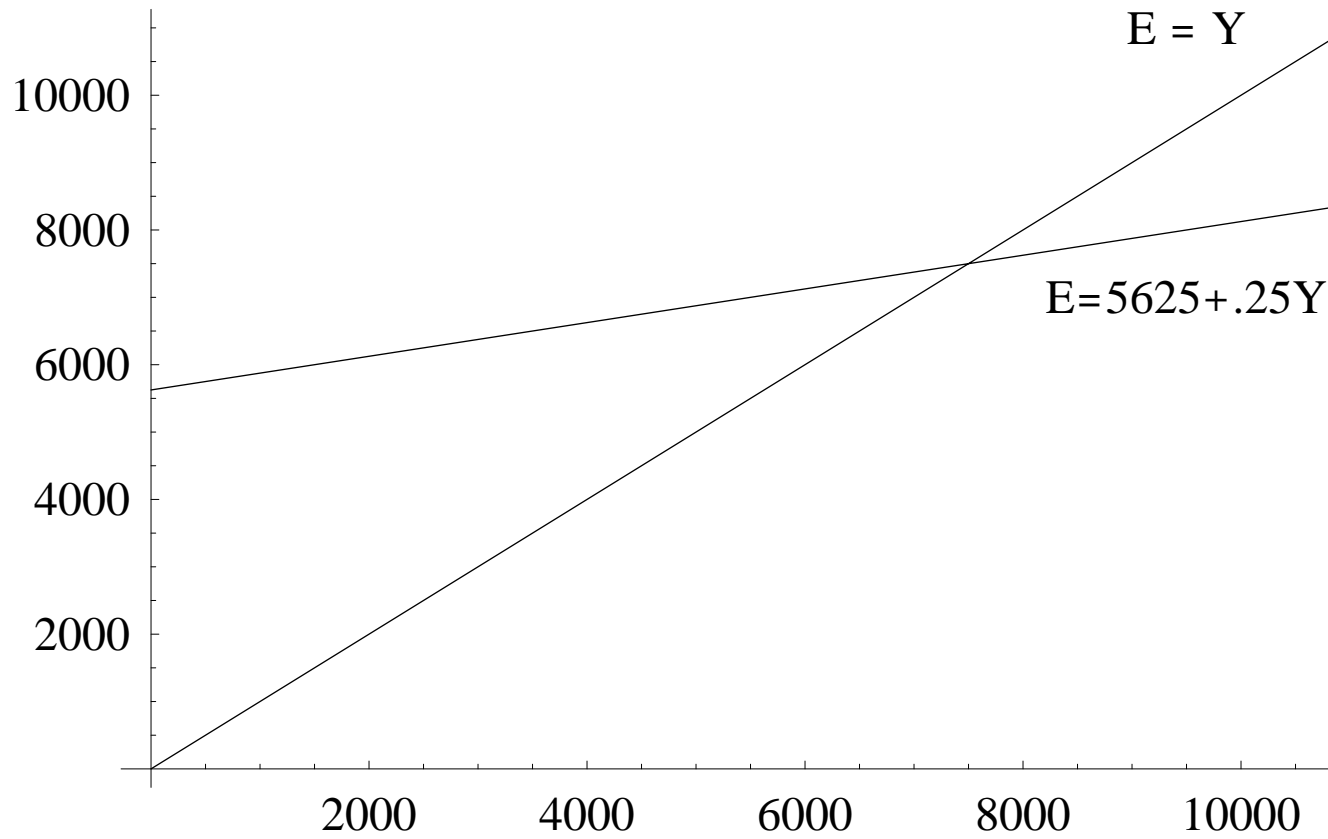


Figure 3

The new slope would be then solve for the new tax rate:

$$m = \frac{8000 - 5625}{8000 - 0}$$

$$0.296875$$

... and the new expenditure function is $E = 5625 + 0.296875Y$

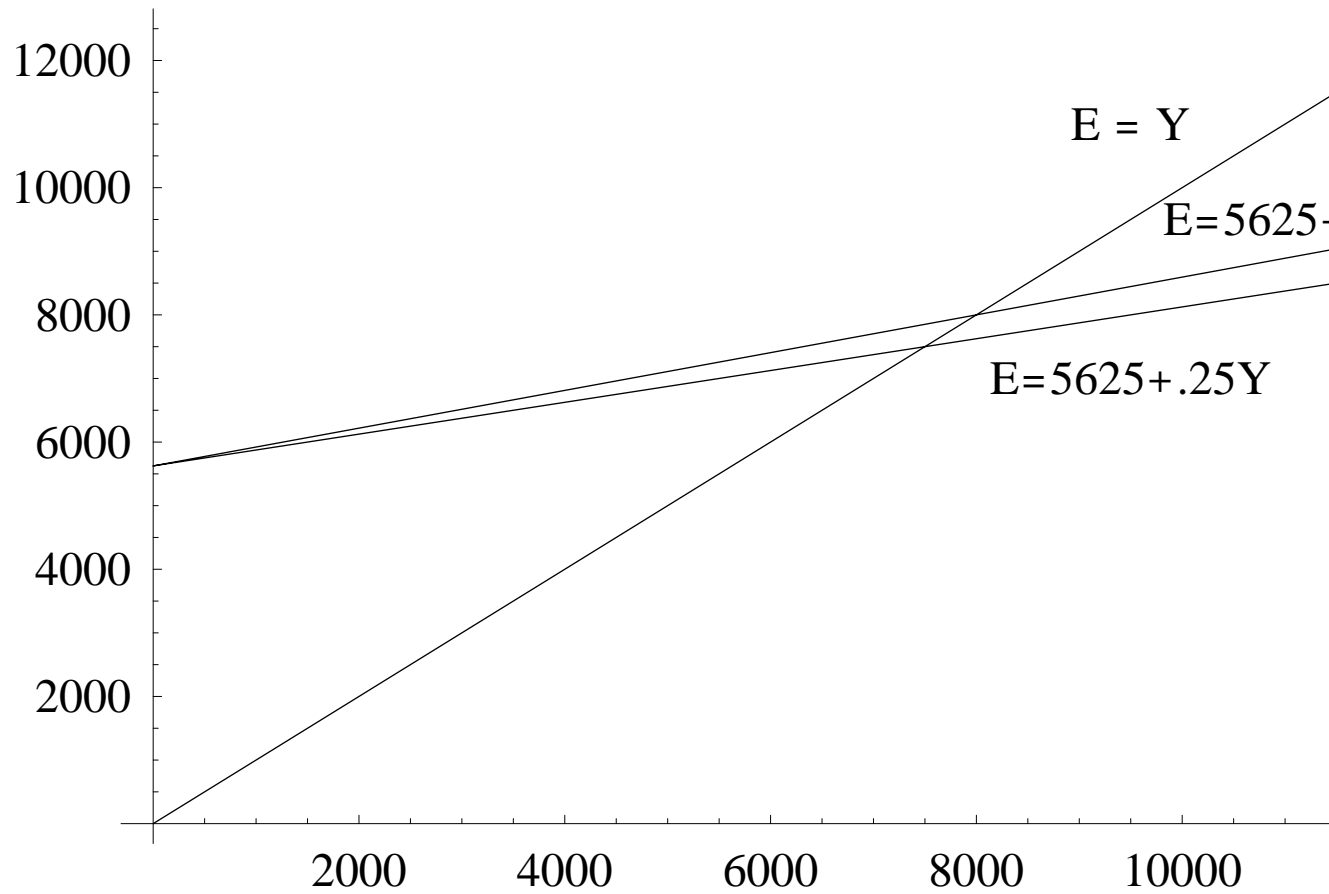


Figure 4

Knowing the slope we want Planned Expenditure to have, it's easy to solve for the needed tax rate:

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Solve [m == c (1 - t1) - μ, t1] [[1, 1]]
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t1 → 0.171875
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This corresponds to an increase in the tax rate of:

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Δt = 0.25 - 0.171875
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0.078125
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The change in total tax revenues would be :

$$(0.171875 * 8000) - (0.25 * 7500)$$

$$-500.$$

You probably noticed that an increase in National Income of a magnitude of 500 required a decrease in tax revenue of exactly the same magnitude. You probably also wonder why... Well it's spurious, i.e. had we started with different parameters to begin with, it wouldn't have happened. The decrease in tax revenues may have turned out to be larger or smaller than the corresponding increase in equilibrium income. In particular is the larger the marginal propensity to consume, the larger the increase in income relative to the decrease in tax revenue. This is intuitive as the marginal propensity is what the effect of a decrease in the tax rate feeds through into consumption.

$$Y[A, c, t, \mu] := \frac{A}{1 - c + c(t) + \mu}$$

Here is the effect of an increase in the tax rate on tax revenue:

$$\frac{A t_0}{1 - c + \mu + c t_0} - \frac{A t_1}{1 - c + \mu + c t_1}$$

Here is the effect of an increase in the tax rate on equilibrium income:

$$-\frac{A}{1 - c + \mu + c t_0} + \frac{A}{1 - c + \mu + c t_1}$$

There is no reason *a priori* for these to be equal to each other. The difference between the resulting increase in equilibrium income and the loss in tax revenues is equal to:

$$-\frac{A}{1 - c + \mu + c t_0} - \frac{A t_0}{1 - c + \mu + c t_0} + \frac{A}{1 - c + \mu + c t_1} + \frac{A t_1}{1 - c + \mu + c t_1}$$

i.e.

$$\frac{A(-1 + 2c - \mu)(t_0 - t_1)}{(1 - c + \mu + c t_0)(1 - c + \mu + c t_1)}$$

this expression will be positive in case of a decrease in the tax rate iff:

$$c > \frac{1 + \mu}{2}$$

In this particular example,

$$c = \frac{1 + \mu}{2} = 0.6$$

b. Suppose the government wants to boost real GDP up to full employment by increasing government spending. How large an increase in government spending is required to boost real GDP to full employment?

To get an increase in national income of a magnitude of 500, one needs an increase in government spending of a magnitude of:

$$\text{NSolve} \left[\Delta Y == \frac{1}{1 - c + c(t) + \mu} \Delta G, \Delta G \right] // \text{Flatten}$$

$$\{\Delta G \rightarrow 375.\}$$

The new intercept is $375 + 5625 = 6000 \Rightarrow E = 6000 + 0.25Y$

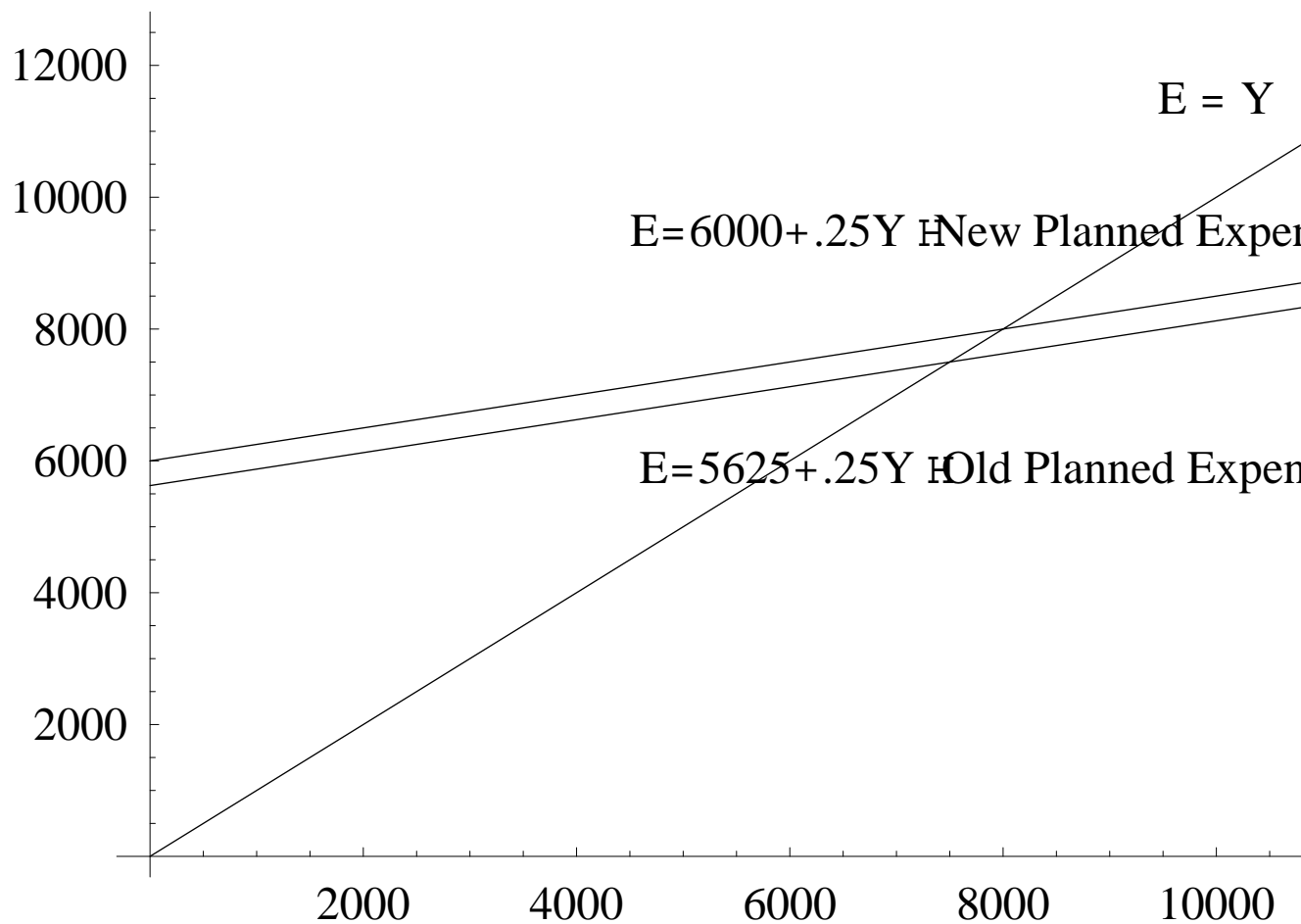


Figure 5

c. Can you account for any asymmetry between the answers to (a) and (b)?

💡 When the government increases its expenditure that directly feeds back into planned expenditure. However, when the government gives a tax break to consumers, they will only spend part of this increase in income (they will save the rest and import foreign goods and services.) Therefore, in this sense, government spending is more effective than tax breaks.

Question 4

a. Suppose that the government and central bank together want to keep GDP constant but raise the rate of investment. What policies can they follow to achieve this

If it wants investment to rise, it could decrease the interest rate (or ask the Fed to do so, possibly under the promise of keeping GDP constant.) But, any increase in investment will result in an increase in equilibrium income (through the multiplier). To prevent this from happening, the government can reduce its expenditure by exactly how much it wants to raise investment.

b. Suppose that the level of investment spending does not depend at all on the interest rate. Does this mean that the IS curve is vertical? If not, how can it be that central bank changes in the real interest rate effect the equilibrium level of real GDP?

It would be vertical if we were in a closed economy. In an open economy, however, an increase in the domestic interest rate appreciates the domestic currency which curtails net exports and it turn (through the multiplier) further raises the equilibrium level of income. This gives some slope to the IS curve even if the level of investment spending does not depend at all on the interest rate.

Question 5

Explain why the IS curve slopes downward. Is its slope steeper in a closed economy--with no international trade--or in an open economy?

The IS curve slopes downward because when the interest rate rises it curtails investment which then results (through the multiplier) in a lower equilibrium income. On top of that an increase in the domestic interest rate appreciates the domestic currency which curtails net exports and it turn (through the multiplier) further raises the equilibrium level of income.

In a closed economy, this second effect is missing. Hence, income is less sensitive to changes in the interest rate. Consequently, the IS curve is steeper in a closed economy than in an open economy.

Question 6

Why does an expansion of government purchases have an amplified impact on the equilibrium level of real GDP? Suppose that the central bank does not target interest rates but instead keeps the money stock constant. Is it still the case that an expansion of government purchases will cause a greater than one-for-one increase in the equilibrium level of real GDP?

An expansion of government purchases has an amplified impact on the equilibrium level of real GDP because of the virtuous circle created by an initial shock to autonomous expenditure, that is because of the multiplier effect (see section notes.) If the central bank does not target interest rates but instead keeps the money stock constant, an increase in income creates excess demand for money which raises the interest rate. This rise in the interest rate partly offsets the increase in equilibrium income generated by the multiplier. Depending on the slope of the LM curve, the expansion of government purchases can cause a greater or smaller than one-for-one increase in the equilibrium level of real GDP. This slope in turn depends on how sensitive the demand for money is to the interest rate.

Question 7

Suppose that the short-term nominal interest rate--the one the central bank actually controls--is 3%. But also suppose that the inflation rate is zero, that the *term premium* is 4%, and that the *risk premium* is 3% as well.

What is the real interest rate relevant for the IS curve?

The real interest rate relevant to the IS Curve is the Long Run Risky interest rate.

Suppose that the IS equation of the economy is: $Y = \$10000 - 300 \times r$. What is the equilibrium level of real GDP?

Since inflation is 0, the short term real interest rate is 3% (So is the nominal).

The Long Run Safe real interest rate: $r^s + \text{term premium} = 3\% + 4\% = 7\%$.

The Long Run Risky real interest rate is the Long Run Safe real interest rate + $\text{risk premium} = 7\% + 3\% = 10\%$

7000

Suppose that the central bank wants to use monetary policy to raise Y to \$9000. Can it do so by open market operations that lower the short-term nominal interest rate? Explain why or why not.

The interest rate that would be needed to set Y to \$9000 is:

3.33333

percent.

However, the lower the Fed can lower the the Long Run Risky real interest rate is 7% because it has no direct control over the term premium and the risk premium. The highest level of equilibrium income it can reach by lowering the interest rate is:

7900

dollar.

What other policy steps can you think of that the government and central bank could take to raise equilibrium real GDP to \$9000?

Increase government spending, cut taxes etc.