

Econ 101b - Answer Key to the Midterm Exam

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Definitions and Concepts

1. Golden Rule savings rate

The savings rate at which consumption per capita is maximized along the steady-state growth path. This saving rate is equal to the share of capital in national income with a Cobb-Douglas production function.

2. Malthusian Trap

A situation where a country experiences a high population growth rate and thus very slowly growing income per capita in spite of technological progress. A country escapes the malthusian trap when technological progress accelerates and finally, demographic transition is achieved.

3. Steady State capital-output ratio

The amount of capital per unit of output along the steady-state growth path. Along the steady-growth path the capital-output ratio is constant; the steady-state capital-output ratio is equal to $s/(n+g+\delta)$.

4. Net exports

The net amount of goods and services sold by American citizens and corporations to foreign customers. Net exports is the difference between gross exports and imports.

5. Permanent / Life-cycle income

The amount of income consumers expect to receive on average over their (remaining) lifetime. It is this income that determines consumption at a deep level.

6. Real interest rate

The nominal interest rate corrected for the rate of inflation. The real interest rate tells a borrower how costly in terms of future purchasing power borrowing is; and tells a lender how profitable in terms of future purchasing power lending is.

7. Federal Reserve

The Federal Reserve, the central bank of the United States, was founded by Congress in 1913 to provide the nation with a safer, more flexible, and more stable monetary and financial system. Today the Federal Reserve's duties fall into four general areas: (1) conducting the nation's monetary policy; (2) supervising and regulating banking institutions and protecting the credit rights of consumers; (3) maintaining the stability of the financial system; and (4) providing certain financial services to the U.S. government, the public, financial institutions, and foreign official institutions. (See <http://www.federalreserve.gov/>)

8. Quantity theory of money

[...] the level of prices varies directly with the quantity of money in circulation, provided the velocity of circulation of that money and the volume of trade it is obliged to perform are not changed [...] (Irving Fisher)

9. Demographic transition

Above a certain level of income per capita, further increases in income per capita decrease the population growth rate instead of increasing it.

10. Macroeconomics

The discipline concerned with the economy as a whole. Macroeconomics studies the behavior of (among others): the interest rate, the exchange rate, the stock market, aggregate consumption, aggregate income. In (neo-keynesian) macroeconomics, markets clear mostly through quantity adjustments rather than price adjustments, at least in the short-run.

Problem 1

Suppose we have country on its steady-state growth path in which output per worker today is \$50,000 per year. Labor force growth is 1.0% per year. The depreciation rate is 3% per year. The rate of growth of the efficiency of labor is 3.0% per year. The rate of growth of the efficiency of labor is 1.0% per year.

Suppose that the economy is on its steady-state growth path, and that the capital output ratio is 4.

a. What is the savings rate?

$$n = 0.01; \delta = 0.03; g = 0.01; kor = 4;$$

The steady-state capital-output ratio:

$$\kappa[s, n, \delta, g] := \frac{s}{n + g + \delta}$$

Has to be set equal to 4 and then we solve for s:

$$0.2$$

b. What is the efficiency of labor if the diminishing-returns-to-investment parameter α equal to 1/2?

$$\left(\frac{Y}{L}\right)_{a,ss} = \kappa^{\frac{\alpha}{1-\alpha}} E_t = 50000$$

Solve for E_t :

12500

c. Suppose further that the country's savings rate were to jump upwards by 5 percentage points. By what proportion would output per worker along the steady-state growth path increase?

$$\frac{\left(\frac{Y}{L}\right)_{t,ss}^b}{\left(\frac{Y}{L}\right)_{t,ss}^a} = \frac{\left(\frac{s_b}{n+g+\delta}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_t}{\left(\frac{s_a}{n+g+\delta}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_t} = \left(\frac{s_b}{s_a}\right)_{ss}^{\frac{\alpha}{1-\alpha}} =$$

$$\left(\frac{0.25}{0.20}\right)^{\frac{\alpha}{1-\alpha}}$$

1.25

$$\left(\frac{Y}{L}\right)_{a,ss} = \left(\frac{K}{Y}\right)_{a,ss}^{\frac{\alpha}{1-\alpha}} E_t = 4.12500 \cdot e^{g^*t}$$

$$\left(\frac{Y}{L}\right)_{b,ss} = \left(\frac{K}{Y}\right)_{b,ss}^{\frac{\alpha}{1-\alpha}} E_t = 5.12500 \cdot e^{g^*t}$$

Output per worker

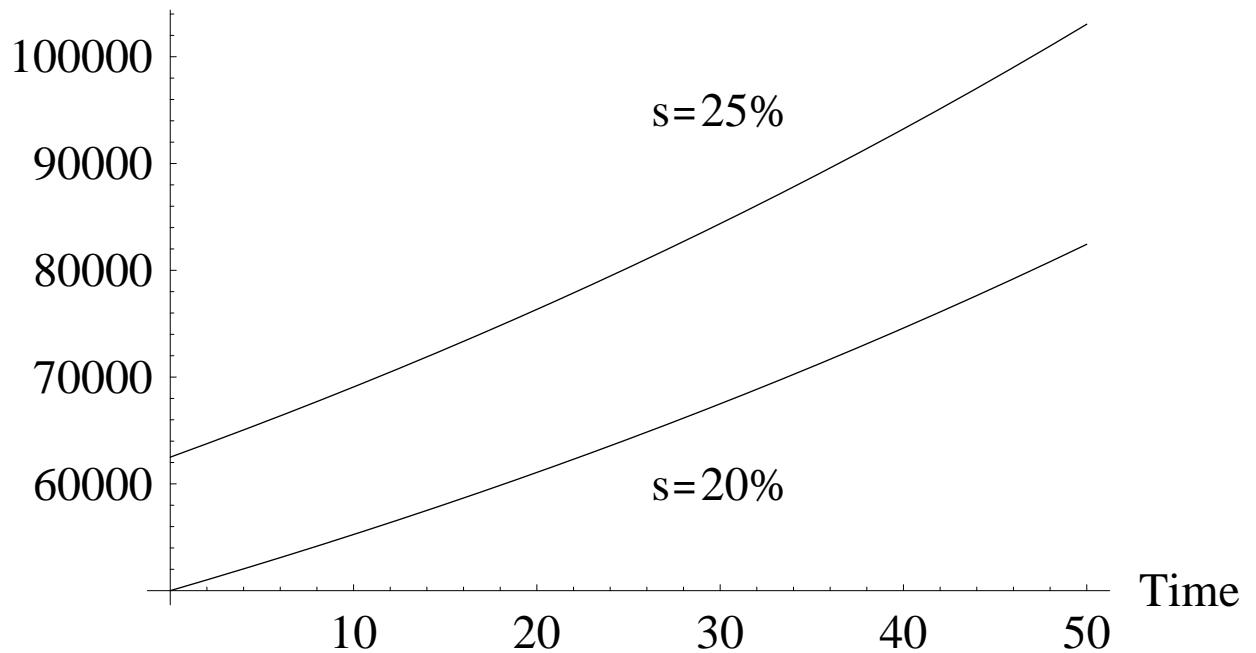


Figure 1

d. You are asked to forecast the effect of such an increase in savings on the level of consumption per worker. Does consumption per worker rise or fall in the year after such an increase in the savings rate? Explain briefly why.

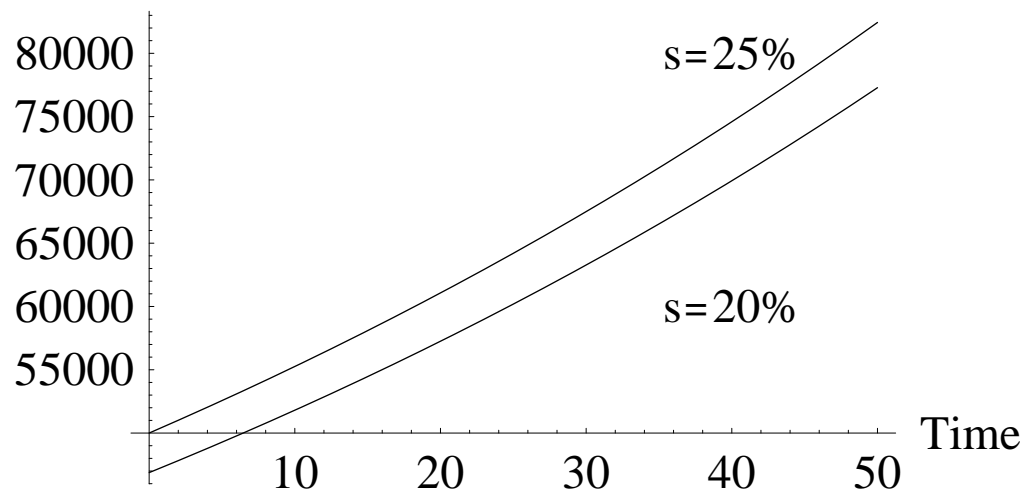
$$\left(\frac{C}{L}\right)_t = (1-s) \left(\frac{K}{Y}\right)_t^{\frac{\alpha}{1-\alpha}} E_t \Rightarrow$$

$$\frac{\left(\frac{C}{L}\right)_{t,ss}^b}{\left(\frac{C}{L}\right)_{t,ss}^a} = \frac{(1-s_b) \left(\frac{s_b}{n+g+\delta}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_t}{(1-s_a) \left(\frac{s_a}{n+g+\delta}\right)_{ss}^{\frac{\alpha}{1-\alpha}} E_t} = \frac{1-s_b}{1-s_a} \left(\frac{s_b}{s_a}\right)_{ss}^{\frac{\alpha}{1-\alpha}}$$

$$\frac{1-0.25}{1-0.20} \left(\frac{0.25}{0.20}\right)^{\frac{\alpha}{1-\alpha}}$$

1.17188

Consumption per worker



e. How many years (approximately) does it take before consumption per worker is above its level along the old steady-state growth path?

$$(1 - \alpha) (n + g + \delta)$$

$$0.025$$

Now,

$$\begin{aligned} \left(\frac{C}{L}\right)_t &= (1-s) \left(\frac{K}{Y}\right)_t^{\frac{\alpha}{1-\alpha}} E_t \Rightarrow \left(\frac{C}{L}\right)_t = (1-s) \frac{\alpha}{1-\alpha} \left(\frac{K}{Y}\right)_t + (1-s) \frac{\dot{E}_t}{E_t} \Rightarrow \left(\frac{C}{L}\right)_t - (1-s)g = \\ & (1-s) \frac{\alpha}{1-\alpha} (1-\alpha)(n+g+\delta) \left(\frac{(K/Y)_{ss} - (K/Y)_t}{(K/Y)_t}\right) \Rightarrow \\ \left(\frac{C}{L}\right)_t - (1-s)g &= (1-s) \alpha (n+g+\delta) \left(\frac{(K/Y)_{ss} - (K/Y)_t}{(K/Y)_t}\right) = \end{aligned}$$

this suggests that near its steady-state growth path an economy closes a very small (and decreasing) fraction of the gap. Thus 2.5% is a gross over-estimate.

Problem 2

Suppose that the consumption and investment functions for an economy is (with all variables in billions of dollars):

$$\text{Cons}[Y_] := 1000 + .75Y;$$

$$\text{Inv}[R_] := 1500 - 10000 * R;$$

$$G = 500;$$

Suppose further that the exchange rate is given by:

$$\varepsilon[R_] := 100 - 100 * (R - 0.10);$$

and that net exports are:

$$\text{NX}[R_] := 1000 - 10 * \varepsilon[R];$$

Suppose, last, that prices are completely flexible, expectations are consistent, the level of potential output given by the production function is:

$$Y = 8000;$$

a. What is the equilibrium level of the real interest rate?

$$\text{Cons}[Y] + \text{Inv}[R] + G + \text{NX}[R] = Y$$

$$900. - 9000. R = 0$$

$$\{R \rightarrow 0.1\}$$

b. What is the equilibrium level of consumption spending?

Just plug in for Y

$$7000.$$

c. What is the equilibrium level of investment?

Just plug in for R

$$\{500.\}$$

d. What is the equilibrium level of net exports?

Just plug in for R in ε

$$\{0.\}$$

Suppose that government purchases were to fall to \$50.

e. What is the equilibrium level of the real interest rate?

$$\text{Cons}[Y] + \text{Inv}[R] + G + \text{NX}[R] = Y$$

$$450. - 9000. R = 0$$

{R → 0.05 }

f. What is the equilibrium level of consumption spending?

7000.

g. What is the equilibrium level of investment?

{1000. }

h. What is the equilibrium level of net exports?

{-50. }

h. Provide, briefly, some economic intuition behind what is going on here.

- ▽ The decrease in government spending increases public saving and thus national saving. This creates excess supply of loanable funds. The interest rate thus decreases which increases investment. What should happen in theory is a depreciation of the currency and an increase in net exports. However, here (it's an error!) the home currency appreciates when the domestic interest rate drops and so, here, net exports decrease.