

NBER WORKING PAPER SERIES

ENGINES OF GROWTH:  
DOMESTIC AND FOREIGN SOURCES  
OF INNOVATION

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Working Paper No. 5207

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
August 1995

This paper presents the authors' own views, and not those of the Federal Reserve System. An earlier version was presented at the Tenth Annual Technical Symposium organized by The Center for Japan-U.S. Business and Economic Studies on April 13, 1995. Poorti Marino and Akiko Tamura provided excellent research assistance, but we remain responsible for any errors. We have benefitted from comments by Charles Himmelberg and other participants at the Technical Symposium. This paper is part of NBER's research programs in Growth, International Finance and Macroeconomics, International Trade and Investment, and Productivity. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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ABSTRACT

We examine productivity growth since World War II in the five leading research economies: West Germany, France, the United Kingdom, Japan, and the United States. Available data on the capital-output ratio suggest that these countries grew as they did because of their ability to adopt more productive technologies, not because of capital deepening *per se*. We present a multicountry model of technological innovation and diffusion which has the implication that, for a wide range of parameter values, countries converge to a common growth rate, with relative productivities depending on the speed with which countries adopt technologies developed at home and abroad. Using parameter values that fit a cross section of data on productivity, research, and patenting, we simulate the growth of the five countries, given initial productivity levels in 1950 and research efforts in the subsequent four decades. Based on plausible assumptions about "technology gaps" that existed among these countries in 1950 we can explain their growth experiences quite successfully. Specifically, the simulations capture the magnitude of the slowdown in German, French, and Japanese productivity growth and the relative constancy of U.K. and U.S. growth.

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# 1 Introduction

The revival of interest about what drives national growth rates has spawned several controversies. One is whether countries that start out poor grow faster than initially rich countries, so that income levels are “converging.” A second is whether sources of growth are primarily domestic or foreign in origin. A third, and perhaps most fundamental, is what causes growth rates in output per worker to differ among countries: differences in capital per worker or differences in available technology.<sup>1</sup>

Whether growth is primarily driven by factor accumulation or by technology, the issue remains as to whether sources of growth are primarily foreign or domestic in origin. If capital accumulation is the key to growth, and international capital markets are highly segmented, countries must rely on their own savings to finance investment. A country with an initially lower level of capital has to finance the investment needed to catch up with its neighbors on its own, which could take a long time. Moreover, a country with a lower savings rate than its neighbors will never catch up, condemned to a permanently lower relative level of output per worker. In contrast, with a high degree of capital market integration a backward country can catch up rapidly by borrowing from abroad. Cross-country differences in savings still imply permanent differences in national levels of GNP per worker, as low savers find themselves in debt to high savers, but levels of GDP per worker

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<sup>1</sup>Differences in human capital may also be crucial to a country's growth prospects, as argued by Lucas (1988) and Barro (1991). One view is that human capital facilitates the adoption of new technology (Benhabib and Spiegel (1993)). An alternative view treats human capital as another factor of production, in which case accumulation of human capital per worker raises effective labor per worker (Mankiw, Romer, and Weil (1992) and Barro, Mankiw, and Sala-I-Martin (1995)). Thus, human capital can play a role in either the technological-adoption explanation or the capital-deepening explanation of differences in productivity growth.

will soon converge.

Taking the alternative view that technological innovation and diffusion drive national growth rates, if innovations are applicable only at home, a country must innovate on its own to raise total factor productivity. A backward country has to be more innovative than its neighbors in order to catch up with them, and catching up is likely to take time. But if innovations are easy to adopt regardless of where they came from, a technologically backward country can catch up rapidly by absorbing the most advanced technologies, and an innovative country gains little relative advantage in terms of factor productivity.<sup>2</sup>

Our purpose here is to examine some evidence on these issues. We begin by showing that capital deepening provides at best an incomplete explanation of the growth in manufacturing productivity of the leading economies since World War II. We then examine what role technological innovation and international diffusion play in explaining why countries grew as they did.

We adopt a specific model of international technology diffusion taken from Eaton and Kortum (1994). Various parameterizations yield special cases with different implications for growth and convergence. In our earlier work we chose parameters by fitting the model to data on productivity, research, and patenting from the five leading research economies (the United States, Japan, Germany, France, and the United Kingdom) for 1988. In doing so we assumed that by then these countries had achieved a steady state in which they were growing on average at a common rate, since by that point their growth rates were fairly similar.

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<sup>2</sup>We do not review these controversies in detail. Romer (1994), Grossman and Helpman (1994), and Barro and Sala-I-Martin (1994) provide excellent discussions. Coe and Helpman (1993), Eaton and Kortum (1994,1995), and Evenson and Englander (1994) examine the empirical content of models of international technology diffusion.

In the current paper we see how well the model explains manufacturing productivity growth in the same five countries from 1950 to 1990, a period that began with the countries growing at very different rates. Since an assumption that these countries were in steady-state throughout the period is inappropriate, we interpret their growth experiences in terms of the out-of-steady-state behavior of the model. The state variables governing the model's dynamics are productivity levels and the pools of ideas from at home and abroad that individual countries have yet to adopt. We initialize our model by setting productivity levels at their actual 1950 values. Of course we do not know the size of the pools of knowledge available to these countries in 1950. We make the simple assumption that the pools are proportional to what they would be if 1950 were a steady state. Using the parameter estimates from our previous paper we calibrate one additional parameter governing the overall level of the pools.

We find that the model predicts growth rates after 1950 that are quite close to actual ones. To fit the post war experience, the pools available to the United States must be only a bit larger relative to U.S. productivity in 1950 than they would be if 1950 were a steady state. This means that these pools are very large relative to productivity levels for Japan and the three European countries, since these countries were much further behind the United States in 1950 than a steady state would dictate. Our model picks up the moderate and relatively constant rate of U.S. productivity growth. It also explains the rapid growth of Japan, Germany, and France in the 1950's and 1960's in terms of the large pools of ideas available to them. As these pools shrink relative to these countries' levels of productivity, growth slows to rates more like that in the United States. Our model also tells this story for the U.K., thus capturing its experience in the first three decades after

World War II. It fails to predict the U.K. growth revival of the 1980s, however.

Is it plausible to think that Japan and Europe had great potential to grow after World War II by adopting foreign technology? We propose two arguments. First, the United States was clearly a technological leader, even before World War II, and was therefore a great source of technology for others to adopt.<sup>3</sup> Second, the war effort itself produced knowledge about how to apply a number of new technologies.<sup>4</sup> Civilian applications of these technologies were left to exploit after the war.

We proceed as follows. Section 2 below examines data on productivity growth and capital deepening in manufacturing for the five leading research economies. Section 3 presents a particular model of technology diffusion and discusses its implications for growth under various parameter values. In section 4 we consider how well the model explains the growth of these countries between 1950 and 1990. We also simulate how the post-war growth experience would change had technology diffusion patterns been different than they were. Section 5 concludes.

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<sup>3</sup>To quote Nelson and Wright (1992), "The process of global diffusion and adoption of American methods would surely have continued, however, either by imitation or by direct foreign investment, if it had not been interrupted by World War II (page 1945)...The United States came out of World War II bouyant, with technological capabilities extended by wartime experience (p. 1950)." Techniques of mass production were a source of U.S. dominance prior to World War II but were at first slow to diffuse. Womack et. al. (1991) claim that "Much of the European economic miracle of the 1950's and 1960's was nothing more than a belated embrace of mass production (pages 234-235)."

<sup>4</sup>Examples include: innovations in aluminum fabrication stemming from war-time production of aircraft in Germany (Peck (1962)), magnetic recording in Germany, guided rockets in Germany, jet engines in England, Radar in the United States, silicone products in the United States, and titanium in the United States (Jewkes, Sawyers, and Stillerman (1969)).

## 2 Productivity and Capital: A Glimpse at the Data

Table 1 summarizes what happened to value added per hour worked in manufacturing in the United States, Japan, Germany, France, and the United Kingdom over the four decades since World War II. We use a measure of productivity in the manufacturing sector for several reasons. First, most innovations are used in manufacturing. Second, we do not want to count productivity growth brought about by labor reallocation from agriculture to manufacturing or from manufacturing to services. Third, the land share in manufacturing is low compared with that in services, so that land availability plays much less of a role in determining labor productivity. We use internationally comparable data compiled at the University of Groningen as part of the International Comparisons of Output and Productivity project.<sup>5</sup>

Several things stand out. During the 40-year period there were only two reversals of relative position: between France and Germany for second place and between the United Kingdom and Japan for last place. Nevertheless, growth rates over the entire period were very different, with Japan at 7.4 per cent, France at 4.9 per cent, Germany at 4.5 per cent, the United Kingdom at 4.1 per cent, and the United States at 2.6 per cent. Germany, Japan, and France experienced a slowdown of growth throughout the period. For the United States and United Kingdom, however, the slowdown in the 1970s was reversed, with the 1980s representing the period of fastest growth. Consistent with the convergence hypothesis,

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<sup>5</sup>Van Ark and Pilat (1993) describe in detail how the data are constructed. We take van Ark's (1995) data on manufacturing value added per hour in each country relative to the United States in 1950, 1970, 1980, and 1990 and multiply these numbers by the U.S. Bureau of Labor Statistic's (1991) data on U.S. manufacturing value-added per hour in each year. The data are normalized by 1990 U.S. productivity. Growth rates are compounded continuously.

Table 1: Value Added per Hour in Manufacturing

year	Germany	France	U.K.	Japan	U.S.
	(levels, relative to the U.S. in 1990)				
1950	.14	.13	.13	.04	.35
1970	.45	.41	.29	.25	.57
1980	.67	.64	.37	.47	.71
1990	.86	.91	.66	.78	1.00
	(annual rates of growth, percent)				
1950-1970	5.9	5.7	3.9	9.0	2.4
1970-1980	4.2	4.3	2.4	6.2	2.2
1980-1990	2.4	3.6	5.8	5.1	3.5
1950-1990	4.5	4.9	4.1	7.4	2.6

*Sources:* Value added per hour in manufacturing in each country relative to the United States is from van Ark (1995). Value added per hour in U.S. manufacturing is from BLS (1991).

the country that led in 1950, the United States, had the lowest growth rate over the entire period while Japan, the country with the lowest productivity in 1950, grew the fastest. The range of growth rates in the most recent period is much smaller, but their ranking is less supportive of convergence. It is true that productivity growth in the two laggards was highest, with U.K. productivity growing at 5.8 per cent and Japanese productivity growing at 5.1 per cent. U.S. productivity growth rose to 3.5 per cent, however, ahead of West Germany, which grew at only 2.4 per cent.

Turning to capital as an explanation of these differences in productivity levels and growth rates, table 2 reports data on the ratios of capital to labor and of capital to value added in manufacturing constructed from van Ark and Pilat (1993) and the U.S. Bureau of Labor Statistics (BLS) (1991) for three of our five countries.<sup>6</sup>

<sup>6</sup>We back out the ratio of capital to labor and capital to value added for Germany and Japan relative to the United States from van Ark and Pilat's (1993) tables of relative total factor productivity levels, the labor share used to construct them, and relative value added per hour. Since the relative value added per hour is not available for 1960, we use a geometric interpolation to estimate it using the available data in 1950, 1955, and 1965. We then scale these using the

Table 2: Capital in Manufacturing

year	capital-labor ratios			capital-value added ratios		
	Germany	Japan	U.S.	Germany	Japan	U.S.
1950	.14		.30	1.01		.87
1955		.06	.35		.86	.87
1960	.19	.05	.41	.74	.61	.96
1973	.55	.28	.60	1.06	.88	.92
1979	.77	.49	.74	1.12	1.10	1.05
1990	.99	.77	1.00	1.15	.99	1.00

Sources: Capital-hours ratios and capital-value added ratios for German and Japanese manufacturing relative to the United States in each year are from van Ark and Pilat (1993). These same ratios for U.S. manufacturing are from BLS (1991).

Again, levels are reported relative to the United States in 1990.

Capital-labor ratios do vary positively with value added per hour worked, both across countries and over time. However, this does not imply that capital deepening was a source of productivity differences. In fact, if capital is internationally mobile then differences in capital-labor ratios are driven by differences in technology, as capital moves to take advantage of its higher marginal product in countries where technology is more advanced.

However, with perfect international capital mobility, Harrod-neutral differences in technology do not affect the ratio of capital to *value added*. In contrast, if differences in labor productivity result from differences in savings rates among countries with identical technologies then higher labor productivity should be associated with a strictly higher ratio of capital to value added. Thus, the behavior of the capital-value-added ratio can distinguish between capital-based and technology-based explanations of productivity differences.<sup>7</sup>

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inverse of an index of value added per unit of capital from BLS (1991). Again these ratios are relative to those in the United States in 1990.

<sup>7</sup>Consider two countries  $r$  and  $p$ , in each of which value added is produced at constant returns to

In fact, as shown in table 2, we find little association between productivity and the capital-value added ratio. Indeed, while the capital-value added ratio has fluctuated, the data largely confirm the Kaldor (1961) view that the capital-output ratio is constant during the process of growth. Consider the variation over time first. While there has been a small increase in capital-value-added ratios, the rate of increase has been about the same in each of the three countries. Thus, capital-deepening does not explain why Japanese productivity grew much faster than U.S. productivity.

Looking across countries in any given year strengthens the argument even further. In 1950 U.S. manufacturing productivity was more than two and a half times Germany's and nearly 8 times Japan's. Assuming a Cobb-Douglas production function with a capital share of 1/3 and identical technology, the U.S. capital-value-added ratio should have been more than 6 times Germany's and more than 60 times Japan's. But the evidence from the nearest years for which we have data indicates differences nowhere near these magnitudes.<sup>8</sup>

While we only examine data for three countries, Japan and Germany are two where pure capital deepening is often held to be an important source of growth af-

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scale. Value added per worker in country  $i$  is given by  $Q_i = F(K_i, A_i)$  where  $F$  is a homogenous function common to the two countries,  $K_i$  is the capital stock per worker in country  $i$ , and  $A_i$  represents the Harrod-neutral level of technology in country  $i$ . Say that  $Q_r > Q_p$ . One possibility is that  $K_r > K_p$  while  $A_r = A_p = A$ . But the diminishing marginal product of capital would then imply that  $\frac{K_r}{F(K_r, A)} > \frac{K_p}{F(K_p, A)}$ , i.e., a strictly higher capital-value added ratio in  $r$ . Another possibility is that  $A_r > A_p$  while capital mobility establishes the condition that  $F_K(K_r, A_r) = F_K(\frac{K_r}{A_r}, 1) = F_K(\frac{K_p}{A_p}, 1) = F_K(K_p, A_p)$ . An implication is that  $\frac{K_r}{A_r} = \frac{K_p}{A_p}$ . In this case, while, as before,  $K_r > K_p$ , i.e., the ratio of capital to physical labor is higher in  $r$ , now  $\frac{K_r}{F(K_r, A)} = \frac{1}{F(1, \frac{A_r}{K_r})} = \frac{1}{F(1, \frac{A_p}{K_p})} = \frac{K_p}{F(K_p, A)}$ , that is, the two countries have the same ratios of capital to value added.

<sup>8</sup>With a much broader set of data, King and Levine (1994) find some evidence that capital-output ratios rise with productivity. Nevertheless, their data require a capital share of at least 2/3 for cross-country differences in this ratio to account for observed productivity differences. Moreover, like Kaldor, they find no evidence that capital-output ratios in individual countries grow as countries become more productive.

ter World War II. From this perspective, the relative constancy of the capital-value added ratios in the manufacturing sectors of all three countries is key evidence against the view that different rates of capital deepening explain why developed countries experienced different growth rates.<sup>9</sup> Instead we turn toward patterns of innovation and diffusion to understand what happened.<sup>10</sup>

### 3 A Model of Innovation and International Technology Diffusion

We now present a multicountry model of international technology diffusion that treats capital as perfectly mobile across countries. Under particular parameter values the model reduces to the following special cases:

**Equal rates of diffusion within and between all countries:** If all countries have access to the same technologies then some other factor is needed to explain differences in productivity across countries, even though technology may be the source of growth in world productivity. An implication, noted by Lichtenberg (1992), is that cross-sectional differences in national levels of productivity should then be unrelated to cross-country differences in national research expenditures. Instead Lichtenberg finds that countries that spend more on research attain higher levels of productivity.

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<sup>9</sup>This evidence is consistent with Mankiw, Romer, and Weil's (1992) finding that productivity is positively correlated with the investment rate. In our view, that correlation is driven by underlying differences in technology.

<sup>10</sup>King and Rebelo (1994) point out the implausibly large interest rate differentials implied by a capital-deepening explanation of productivity differences. Barro, Mankiw, and Sala-i-Martin (1995) address this issue by introducing internationally immobile human capital as a third factor of production. Nonetheless, their model still implies that the capital-output ratio should rise with productivity.

**No diffusion between countries:** At the opposite extreme, each country might use only those technologies discovered at home. In this case, except by coincidence, levels of productivity across countries will diverge. Endogenous growth models in which a country's productivity depends only on its own rate of investment in research have this implication. This implication is not, however, consistent with evidence that relatively backward countries make use of the technologies of their more advanced neighbors, i.e. "technological catch-up".<sup>11</sup> It also fails to explain why inventors frequently take out patents in a number of different countries.

**A common pool of technology adopted at different rates in different countries:** A more general possibility is that all research outcomes enter a common pool which individual countries can tap. Countries may differ, however, in their ability to draw on this pool.<sup>12</sup> Under this specification, countries' *levels* of productivity converge to a common level only if they exploit the common pool at the same rate. Even if they do not, however, national *growth rates* of productivity will converge if a more backward country finds that a greater fraction of the ideas in the pool are worth adopting. An advantage of this approach is that it can explain steady-state differences in productivity levels while technological spillovers nevertheless occur. A deficiency is that, like equal diffusion, it implies that a greater national research effort confers no relative national advantage, contrary to

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<sup>11</sup>Economic historians have appealed to this notion to explain the spread of the industrial revolution in the nineteenth century. See, for example, Gershenkron (1962) and Fagerberg (1994). Nelson and Phelps (1966) provide an early model of diffusion from an advanced to a backward country. Helliwell and Chung (1991) report recent evidence supporting the view that low-productivity countries benefit from high-productivity ones.

<sup>12</sup>Parente and Prescott (1994) take this approach in relating a country's level of productivity to its willingness (promoted by low tax rates) to adopt new technologies from around the world. Benhabib and Spiegel (1993) also take this approach in a model in which high levels of human capital promote adoption.

Lichtenberg's (1992) results.

**Diffusion rates depend on the countries involved:** The most general specification makes the rate of diffusion specific to the source and destination of the innovation. If it is the case that diffusion is more rapid within than between countries, then a country can attain a higher relative level of productivity by doing more research. Faster adoption of technology also raises productivity. Productivity growth rates again may be equalized since backward countries have a larger backlog of ideas to adopt.<sup>13</sup>

Our model contains the following components:

### 3.1 Production

Output in country  $n$  is produced by combining intermediate inputs subject to a constant-returns-to-scale Cobb-Douglas production function,

$$Y_{nt} = \exp \left( \int_0^1 \ln[Z_{nt}(j)X_{nt}(j)]dj \right),$$

where  $X_{nt}(j)$  is the quantity of input  $j$  produced at time  $t$  in country  $n$  and  $Z_{nt}(j)$  is the quality of that input. There are  $n = 1, \dots, N$  countries. They trade the homogeneous output but not the inputs.<sup>14</sup> To produce any input at rate  $x$  requires labor services at rate  $x$ .<sup>15</sup> As in Grossman and Helpman (1991), productivity differences across countries arise solely from differences in the quality of inputs.

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<sup>13</sup>This is the case considered in Eaton and Kortum (1994, 1995), Park and Brat (1995), and Ben-David and Loewy (1995).

<sup>14</sup>By assuming that inputs are not traded, we ignore some interesting terms-of-trade effects as are illustrated in Johnson and Stafford (1993).

<sup>15</sup>More generally,  $z$  could be provided by a combination of (nontraded) labor and (internationally mobile) capital. We would require, however, that these two factors combine to provide  $z$  homothetically in a way that did not vary across inputs. For simplicity, however, we will refer to  $z$  as the labor input.

Within a country the quality of inputs improves over time as a consequence of research performed domestically and abroad. We use the index,

$$A_{nt} = \exp \left( \int_0^1 \ln Z_{nt}(j) dj \right).$$

to aggregate the qualities of individual inputs into a single measure of overall labor productivity.<sup>16</sup>

### 3.2 Ideas

Quality improvements result from new ideas. There are three dimensions to an idea: (i) its quality, (ii) its sector of application, and (iii) the time until it diffuses to each country.

An idea's quality is a random variable  $Q$  drawn from the Pareto distribution,

$$F(q) = \Pr[Q < q] = 1 - q^{-\theta}.$$

A given idea has the same quality no matter where it is used.

An idea applies only to one out of the continuum of inputs. The input  $j$  to which the idea applies is drawn from the uniform distribution on  $[0, 1]$ .

An idea takes time to learn about and to apply to a specific purpose. If an idea is discovered at time  $t$  in country  $i$  then it diffuses to country  $n$  at time  $t + \tau_{ni}$ , for  $n = 1, 2, \dots, N$ . We assume that the marginal distribution of the diffusion lag from country  $i$  to country  $n$  is exponential with parameter  $\epsilon_{ni}$ , i.e.

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<sup>16</sup>This index actually equals labor productivity if production workers are evenly divided among production of the individual inputs, as would occur if labor were allocated by a central planner. In our model, however, different inputs are subject to different degrees of monopoly power, so that labor is allocated unequally across inputs. Nevertheless, as we show in Eaton and Kortum (1994), labor productivity is proportional to this index in a decentralized equilibrium.

$\Pr[\tau_{ni} \leq x] = 1 - e^{-\epsilon_{ni}x}$ .<sup>17</sup> Thus  $\epsilon_{ni}$  is the speed of diffusion from country  $i$  to country  $n$  and  $\epsilon_{ni}^{-1}$  is the mean diffusion lag. As  $\epsilon_{ni}$  goes to infinity diffusion becomes instantaneous while as  $\epsilon_{ni}$  goes to 0 ideas from country  $i$  are never adopted in country  $n$ .

Different restrictions on the  $\epsilon_{ni}$ 's capture the various special cases of diffusion that we discussed above. If there is equal diffusion among all countries then  $\epsilon_{ni} = \epsilon$  for all  $n, i$ . No diffusion between countries means that  $\epsilon_{ni} = 0$  for  $n \neq i$ . If countries draw on a common pool at different rates then,  $\epsilon_{ni} = \epsilon_n \epsilon_{.i}$ , where  $\epsilon_n$  reflects country  $n$ 's ability to absorb technology and  $\epsilon_{.i}$  reflects country  $i$ 's ability to make it available. In general, however, diffusion rates can depend on the specific countries.

We assume that country  $i$  produces new ideas at rate  $E_{it} = \alpha_{it} L_{it}^{1-\beta} R_{it}^\beta$  where  $R_{it}$  is research employment in country  $i$ ,  $L_i$  is the workforce there (including researchers), and  $\alpha_i$  and  $0 \leq \beta \leq 1$  are parameters governing the productivity of researchers. If  $\beta < 1$  then countries face decreasing returns in the intensity with which they perform research.<sup>18</sup>

At any time  $t$  country  $n$  can draw upon the pool of unexploited ideas from country  $i$ . As country  $i$  undertakes research it generates a flow into this pool of  $E_{it}$  while, as country  $n$  adopts them, it depletes the pool in proportion to its size at rate  $\epsilon_{ni}$ . We introduce  $N^2$  state variables,  $\eta_{ni}$ , representing the stock of ideas

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<sup>17</sup>The distribution of the diffusion lags across destination countries need not be independent. Hence if a particular invention is absorbed particularly quickly by country  $n$  then it might be more likely to be absorbed quickly by country  $m$  as well.

<sup>18</sup>Let  $U$  be the an individual's talent for research. We imagine  $U$  being drawn from a probability distribution,  $\Pr(U \leq u) = G(u)$ . Suppose that a researcher with talent  $U$  has ideas at rate  $\alpha\beta U$ . If individuals with the most talent for research actually become researchers then the talent of the least talented researcher solves  $1 - G(u^*) = R/L$ . The aggregate rate of idea production is therefore,  $\alpha\beta L \int_{u^*}^{\infty} u dG(u)$ . We obtain the functional form used in our model if the talent distribution is Pareto,  $G(u) = 1 - u^{-\frac{1}{1-\beta}}$ .

from country  $i$  that have not yet diffused to country  $n$ . Since  $\dot{\eta}_{nit} = E_{it} - \epsilon_{ni}\eta_{nit}$ ,

$$\eta_{nit} = \int_{-\infty}^t e^{-\epsilon_{ni}(t-s)} E_{is} ds$$

is the size of the pool of ideas from country  $i$  that country  $n$  has not yet drawn upon. Finally, we introduce  $N$  state variables,  $\mu_n$ , representing the stock of ideas that have diffused to country  $n$ . Ideas flow to country  $n$  from the stocks of undiffused ideas,

$$\dot{\mu}_{nt} = \sum_{i=1}^N \epsilon_{ni} \eta_{nit}, \quad (1)$$

where  $\mu_{nt} = \int_{-\infty}^t \dot{\mu}_{ns} ds$ .

### 3.3 The Technological Frontier

We distinguish between the concepts of diffusion and adoption. While every idea from country  $i$  will eventually diffuse to country  $n$ , some ideas will not be adopted. Only the best available idea for each input in each country is actually used. Thus, for each country  $n$ ,  $Z_{nt}(j)$  represents the highest quality idea that has diffused to country  $n$  in sector  $j$  by time  $t$ . A new idea diffusing there will be adopted if and only if its quality exceeds  $Z_{nt}(j)$ .

The technological frontier in country  $n$  at time  $t$  represents the quality of the ideas being used in each sector. The position of this frontier is conveniently summarized by a distribution function,  $H_n(z|t)$ , representing the fraction of sectors with quality below  $z$ . As we show in the appendix, this distribution is given by:

$$H_n(z|t) = e^{-\mu_{nt} z^{-\theta}} \quad (2)$$

which depends only on the stock of diffused ideas  $\mu_{nt}$  regardless of when these ideas were adopted for production or where they came from.

An idea of quality  $q$  that has diffused to country  $n$  will be adopted there with probability  $H_n(q|t)$ . Therefore, integrating over the probability density of possible qualities  $F'(q) = \theta q^{-(\theta+1)}$ , we obtain the probability of adoption,

$$\int_1^\infty H_n(q|t)F'(q)dq = \mu_{nt}^{-1}.^{19}$$

We define the inventive step of a newly diffused idea as the percentage improvement in quality it brings about if adopted. The average inventive step of a new idea, conditional on its adoption, is  $\int_z^\infty \ln\left(\frac{q}{z}\right) \frac{F'(q)}{1-F(z)} dq = \theta^{-1}$ .

Productivity growth in country  $n$  is simply the product of three terms: (i) the rate of arrival of newly diffused ideas, (ii) the probability that an idea will be adopted, and (iii) the average inventive step of the ideas that are adopted. Combining our expressions for each,  $\frac{\dot{A}_{nt}}{A_{nt}} = \frac{1}{\theta} \frac{\dot{\mu}_{nt}}{\mu_{nt}}$ . Integrating this equation yields the following relationship between our index of productivity  $A_{nt}$  and the stock of ideas that have arrived in country  $n$  by time  $t$ :

$$A_{nt} = c\mu_{nt}^{1/\theta}, \tag{3}$$

where  $c$  is a constant derived in the appendix.

A country with a lower level of productivity adopts a higher proportion of the ideas coming its way,  $\mu_{nt}^{-1} = c^\theta A_{nt}^{-\theta}$ . In this sense the model captures the idea of technological catch-up whereby a country with a lower level of productivity can grow faster by taking advantage of the ideas of others. Indeed, given the rate at

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<sup>19</sup>Here and below we use approximations that become arbitrarily close (in percentage terms) as  $\mu$  becomes large. See the appendix.

which ideas diffuse into a country, its productivity growth is inversely related to its level of productivity,

$$\frac{\dot{A}_{nt}}{A_{nt}} = \frac{c^\theta}{\theta} \dot{\mu}_{nt} A_{nt}^{-\theta}.$$

However, one country's level of productivity may continue to lag behind another's if fewer ideas diffuse to it. We now turn to the dynamics of this diffusion process.

### 3.4 Productivity Dynamics

The state of the whole system at any time  $t$  can be summarized by the  $N^2 + N$  vector of state variables

$$y_t = [\eta_{11t}, \dots, \eta_{1Nt}, \dots, \eta_{N1t}, \dots, \eta_{NNt}, \mu_{1t}, \dots, \mu_{Nt}]',$$

representing the size of the  $N^2$  pools of unexploited ideas and the size of the  $N$  stocks of ideas that have diffused. The system of differential equations:

$$\begin{aligned} \dot{\mu}_{nt} &= \sum_{i=1}^N \epsilon_{ni} \eta_{nit}, \\ \dot{\eta}_{nit} &= E_{it} - \epsilon_{ni} \eta_{nit}, \end{aligned} \tag{4}$$

describe how the vector of state variables evolves over time given paths for the forcing variables,  $E_{i,s}$  (reflecting research effort and the work force in each country) and the  $N^2 + N$  initial conditions for the state variables.

We consider two cases for how the productivity of researchers (in coming up with ideas) varies across countries or over time. The first case sets  $\alpha_{it} = \alpha$  as in Kortum (1995). The second case sets  $\alpha_{it} = \alpha \mu_{it}$  as in Eaton and Kortum (1994).

In the second case, researcher productivity is proportional to the stock of ideas that have diffused to a country. Over time, the productivity of researchers rises as the stock of ideas grows. In either case, we get a system of linear differential equations. For each case we provide assumptions that give a steady-state with parallel growth, i.e., in which productivity growth is constant over time and equal across countries.

### 3.5 The case of constant researcher productivity

Consider case 1 first since it is easier to solve. In this case each of the stocks of undiffused ideas can be solved independently given initial conditions at time 0:

$$\eta_{nit} = e^{-\epsilon_{ni}t} \eta_{ni0} + \alpha \int_0^t e^{-\epsilon_{ni}(t-s)} E_{is} ds.$$

while the stocks of diffused ideas are simply:

$$\mu_{nt} = \mu_{n0} + \sum_{i=1}^N \epsilon_{ni} \int_0^t \eta_{nis} ds.$$

Suppose that  $E_{is}$  is constant over time in each country while  $\epsilon_{ni} > 0$  for all  $n, i$ . Stocks of undiffused ideas would approach constants  $\eta_{nit} = \alpha \frac{E_i}{\epsilon_{ni}}$  and hence  $\dot{\mu}_{nt} = \alpha \sum_{i=1}^N E_i$ . Thus, as long as ideas diffuse between all countries (even if at different rates) productivity converges to the same level in each country, regardless of differences in research activity among them. However, if ideas do not diffuse between countries at all then productivity levels in each country converge to levels that depend on that country's research activity. In either case, productivity growth approaches zero.

To sustain growth, the rate of idea production must grow. Say that the workforce in each country grows at rate  $g$  while the fraction of the labor force engaged in R&D remains constant. In this case the stocks of diffused ideas are:

$$\mu_{nt} = \mu_{n0} + \sum_{i=1}^N \eta_{ni0}(1 - e^{-\epsilon_{ni}t}) + \sum_{i=1}^N \frac{\epsilon_{ni}/g}{\epsilon_{ni} + g} \left( 1 - \frac{\epsilon_{ni} + g}{\epsilon_{ni}} e^{-gt} + \frac{g}{\epsilon_{ni}} e^{-(\epsilon_{ni}+g)t} \right).$$

In the long run, with  $g > 0$ , the  $E_{it}$  eventually swamp the initial conditions and the stocks of diffused ideas all grow at rate  $g$  with relative levels given by:

$$\mu_t = \Gamma E_t,$$

where,  $\Gamma_{ni} = \frac{\epsilon_{ni}/g}{\epsilon_{ni}+g}$  and  $E_t = [E_{1t}, \dots, E_{nt}]'$ .

In general, even with no diffusion of ideas between countries, we obtain parallel productivity growth, as all countries eventually grow at the same rate,  $g/\theta$ . Countries that get ideas more quickly, especially from countries that produce the most ideas, will have the highest productivity levels. If there is a world idea pool then relative productivity does not depend on who does research but only on who taps the world pool most rapidly. If all diffusion rates are the same, productivity levels are equal.

### 3.6 The case of rising researcher productivity

In case 2 we must solve the differential equations 4 as a system. Define  $\bar{E}_{it} \equiv \alpha R_{it}^\beta L_{it}^{1-\beta}$ , so that:

$$\dot{\eta}_{nit} = \bar{E}_{it}\mu_{it} - \epsilon_{ni}\eta_{nit}.$$

Then the system of differential equations is,

$$\dot{y}_t = \Delta_t y_t, \quad (5)$$

where,  $y$  was defined above as the  $N^2 + N$  vector of state variables and,

$$\Delta_t = \begin{bmatrix} -\epsilon_{11} & & & & 0 & \bar{E}_{1t} & & 0 \\ & \ddots & & & & & \ddots & \\ & & -\epsilon_{1N} & & & 0 & & \bar{E}_{Nt} \\ & & & \ddots & & & \vdots & \\ & & & & -\epsilon_{N1} & & \bar{E}_{1t} & 0 \\ & & & & & \ddots & & \\ 0 & & & & & & & -\epsilon_{NN} & 0 & \bar{E}_{Nt} \\ \epsilon_{11} & \cdots & \epsilon_{1N} & & & 0 & 0 & & 0 \\ & & & \ddots & & & & & \ddots & \\ 0 & & & & \epsilon_{N1} & \cdots & \epsilon_{NN} & 0 & & 0 \end{bmatrix}.$$

If the  $\bar{E}_{it}$  were to forever grow at a strictly positive rate then growth in the stocks of diffused ideas would eventually accelerate and productivity growth would approach infinity. In the subsequent section we report simulations of the model given the actual paths of total and research employment. For now, however, consider what happens if we set  $\bar{E}_{it} = \bar{E}_i$ . With constant employment of researchers and others, the matrix in the differential equation is constant as well, so that  $\Delta_t = \Delta$ .

Under certain restrictions on the  $\epsilon_{ni}$ 's, which we discuss below, the system converges to a steady state in which all the state variables grow at the same rate,  $g$ . Thus,  $gy_t = \Delta y_t$  where  $g$  is the largest eigenvalue of  $\Delta$ . Thus, the system displays parallel productivity growth. Note that in this version of the model world growth is endogenously determined, depending on research activity in each

country and the rate at which ideas spread between countries.

Given the value of  $g$ , we can analyze a simpler  $N$ -dimensional system to gain some intuition about what determines relative productivity levels. In steady state,  $\eta_{nit} = \frac{\bar{B}_i \mu_{it}}{g + \epsilon_{ni}}$ . Therefore, the vector of relative stocks of diffused ideas,  $\bar{\mu}$ , must satisfy,

$$g\bar{\mu} = \Lambda\bar{\mu},$$

where the matrix  $\Lambda$  has elements  $\Lambda_{ni} = \frac{\epsilon_{ni}\bar{B}_i}{g + \epsilon_{ni}}$ . A country's relative productivity depends on the rate at which it gets ideas, particularly from those countries producing the most ideas. As in the previous version of the model, if  $\epsilon_{ni} = \epsilon_n \cdot \epsilon_i$  then productivity outcomes do not depend on where research is done. And, if diffusion rates within and between all countries are equal, productivity levels will be equal.

What the restrictions on the  $\epsilon_{ni}$ 's ensure that countries converge to parallel growth? Frobenius' theorem guarantees that if the matrix  $\Lambda$  is *indecomposable* then its largest eigenvalue is positive and has associated with it a strictly positive eigenvector. This eigenvalue, the Frobenius root, is the growth rate to which the system eventually converges and the associated eigenvector (defined up to a scalar multiple) gives the relative levels to which the state variables (the pools of diffused and undiffused ideas) converge.

In order for  $\Lambda$  to be indecomposable, no rearrangement of its rows or columns allows it to be represented as block semidiagonal, i.e., it must be impossible to switch columns or rows to write it as:

$$\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{22} \end{bmatrix}$$

where  $\Lambda_{11}$  and  $\Lambda_{22}$  are square matrices. Say that  $\Lambda$  could be represented this way. Then the block of countries corresponding to  $\Lambda_{22}$  is isolated from the rest and (assuming that  $\Lambda_{22}$  is indecomposable) we could solve for its Frobenius root (call it  $\lambda_2$ ) which would determine the rate at which the isolated block of countries would eventually grow. Consider then the block of countries corresponding to  $\Lambda_{11}$ . Assuming that  $\Lambda_{11}$  is indecomposable then, left on their own, these countries would eventually converge to a growth rate determined by the Frobenius root of  $\Lambda_{11}$  (call it  $\lambda_1$ ). One possibility is that  $\lambda_1 < \lambda_2$ , meaning that the isolated set of countries on its own would grow faster than the other block on its own. If  $\Lambda_{12}$  contains strictly positive elements then the “technology gap” between the two blocks would eventually grow to a point at which the isolated block would boost the growth rate of the other countries;  $\lambda_2$  would then determine the rate at which the whole world would eventually grow. We could think of this isolated block, then, as the “engine of growth” for the entire economy. If  $\Lambda_{12} = 0$ , however, then both blocks are isolated, and converge to the growth rates determined by their respective Frobenius roots. This is also the outcome if  $\lambda_1 > \lambda_2$  since in this case the isolated block eventually becomes so small relative to the connected block that it does not influence growth in the connected block. The implication, then, is that the world converges to a common growth path if there are sufficient direct or indirect spillovers among all countries.<sup>20</sup>

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<sup>20</sup>An extreme example of isolation occurs if diffusion within countries is instantaneous while across countries it is nonexistent. In this case the stock of diffused ideas in country  $n$  grows at rate  $g_n = E_n$ , hence productivity growth is  $E_n/\theta$ . As in many models of endogenous growth, a country’s productivity growth would then depend on only its own level of research activity.

## 4 Numerical Simulations of the Model

We now simulate the model to see how well it fits the post-war productivity performance of Germany, France, the United Kingdom, Japan, and the United States (shown above in table 1). We use the version of the model with rising researcher productivity because our parameters, taken from Eaton and Kortum (1994), were chosen under that assumption. In that paper, we chose parameters so that the steady-state of the model matched data on productivity levels, research employment and international patent applications in 1988.

Our estimates of the  $\epsilon_{ni}$  were based on patent applications by inventors from each of the five countries taken out in each of the five.<sup>21</sup> To economize on parameters we restrict  $\epsilon_{ni} = \epsilon_n \epsilon_{iH}$ , where  $\epsilon_n$  reflects country  $n$ 's ability to absorb technology,  $\epsilon_{iH}$  reflects country  $i$ 's ability to provide it, and  $\epsilon_H \geq 1$  allows for faster diffusion to the home country if  $n = i$  ( $\epsilon_H = 1$  if  $n \neq i$ ).

The parameter of the Pareto search distribution is,  $\theta = 1.6$ , the parameter of the research talent distribution is,  $\beta = .012$ , the researcher productivity parameter is  $\alpha = .00088$ , and the diffusion parameters,  $\epsilon_{ni}$ , are:<sup>22</sup>

Destination	Source Country				
	Germany	France	U.K.	Japan	U.S.
Germany	.625	.013	.055	.035	.005
France	.050	.097	.043	.027	.004
U.K.	.021	.004	.178	.011	.002
Japan	.032	.006	.028	.170	.003
U.S.	.037	.007	.032	.020	.029

<sup>21</sup>Eaton and Kortum (1994) incorporate the decision to patent based on: (i) the distribution of the diffusion lag, (ii) the inventive step, (iii) market size, and (iv) the strength of intellectual property protection. Taking the last three factors into account we infer the first.

<sup>22</sup>We actually take  $\alpha/J$  rather than  $\alpha$  from Eaton and Kortum (1994), where  $J$  is a parameter that determines the range of inputs used in a country (in the present paper we normalized  $J$  to unity). The matrix below is based on  $\epsilon_{.1} = .037$ ,  $\epsilon_{.2} = .0074$ ,  $\epsilon_{.3} = .032$ ,  $\epsilon_{.4} = .020$ ,  $\epsilon_{.5} = .0030$ ,  $\epsilon_{1.} = 1.73$ ,  $\epsilon_{2.} = 1.34$ ,  $\epsilon_{3.} = .57$ ,  $\epsilon_{4.} = .87$ , and  $\epsilon_H = 9.77$ .

We take 1950-1990 data on workers (in millions) in each country from Summers and Heston (1991).<sup>23</sup> Although research employment was endogenous in Eaton and Kortum (1994), here we condition on the actual path of research. Hence we are examining how well the model predicts productivity conditional on the R&D that actually occurred, and not how well the model predicts R&D itself.

We take data on business enterprise employment of R&D scientists and engineers from the 1992 OECD STIU Data Base.<sup>24</sup> We then index the data on researchers in each country to the level of 1988 research employment predicted in Eaton and Kortum (1994). That predicted level of research employment is approximately equal to actual research employment, scaled by the fraction of R&D performed by businesses that is privately funded in each country.<sup>25</sup>

The 1988 values of workers and researchers (in millions) are given below.

	Germany	France	U.K.	Japan	U.S.
Workers	29	25	28	61	120
Researchers	.103	.042	.072	.301	.437

If these values were to remain constant over time then the steady state growth rate and relative levels of productivity are easily determined from the largest eigenvalue and the last 5 elements of the associated eigenvector of the matrix,  $\Delta_{38}$  (we count time beginning with 1950 as  $t = 0$ , hence 1988 is  $t = 38$ ). From this we replicate

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<sup>23</sup>We extend the worker data through 1989 and 1990 using an updated version of the Summers and Heston dataset.

<sup>24</sup>Missing OECD data on research employment are interpolated. In the United States and Germany in 1990, we assume that employment of researchers grows at the same rate as the workforce. For the 1950's in the United States we use data from *Employment of Scientists and Engineers: 1950-1970*, Bureau of Labor Statistics, 1972. In the other countries, we extrapolate back to fill in missing data during the 1950's and early 1960's (in Germany, the OECD series begins in 1967, in France, 1964, in the United Kingdom, 1968, and in Japan, 1963).

<sup>25</sup>We do not account for changes in the fraction of privately funded R&D. In the United States, this fraction has had an upward trend.

the fitted values in Eaton and Kortum (1994): the common productivity growth rate is 3.6 per cent and relative productivity levels are, Germany = .92, France = .84, U.K.= .57, Japan = .89, (and U.S. = 1). In this paper, we also calculate the normalized steady-state value of the vector  $\eta$ . We report it below as a matrix with elements  $\eta_{ni}/\mu_n$  representing the stock of ideas from country  $i$  that have not yet diffused to country  $n$  relative to the stock of ideas that have diffused to  $n$  :

Destination	Source Country				
	Germany	France	U.K.	Japan	U.S.
Germany	.04	.25	.09	.52	1.78
France	.27	.13	.12	.67	2.13
U.K.	.66	.61	.10	1.49	4.06
Japan	.29	.29	.13	.22	1.97
U.S.	.23	.24	.10	.54	1.14

The stocks of undiffused ideas vary by column according to the rate of idea production in the source country (thus they tend to be large when the source country is Japan or the United States). They are small on the diagonals since ideas diffuse most rapidly to the home country. Finally, the third row is relatively large since we estimate that the United Kingdom absorbs ideas slowly.

To simulate a dynamic path of productivity we need initial conditions (in 1950) for the vector of stocks of diffused ideas,  $\mu$ , and the vector of stocks of undiffused ideas  $\eta$ , which together constitute  $y_0$ . Obtaining initial conditions for  $\mu$  is straightforward: We solve for them from actual 1950 productivity levels using our estimate of  $\theta$ . How to initialize the vector of stocks of undiffused ideas is less obvious. We set the *relative* values of the elements of  $\eta$  equal to the values determined by a steady state in 1950. We then scale the  $\eta$  vector in order to

maximize the fit of the model, i.e. we minimize,

$$\sum_{n \in C} \sum_{t \in T} (\ln A_{nt} - \ln \hat{A}_{nt})^2,$$

where  $C = \{Germany, France, U.K., Japan, U.S.\}$ ,  $T = \{1950, 1970, 1980, 1990\}$ , and  $\hat{A}$  is productivity simulated from the model, as described below.

The resulting initial conditions are presented below as a matrix with elements  $\eta_{ni}/\mu_n$ .

Destination	Source Country				
	Germany	France	U.K.	Japan	U.S.
Germany	.16	1.43	.59	1.87	6.56
France	1.22	.60	.71	2.16	7.01
U.K.	1.77	1.76	.27	2.77	7.37
Japan	9.53	10.62	5.40	4.51	45.67
U.S.	.30	.35	.17	.51	.97

For the United States these stocks are about equal to the steady state values shown above. The other countries have much greater stocks of undiffused ideas relative to stocks of diffused ideas. These initial conditions capture the hypothesis, discussed in the introduction, that Japan and the European countries entered the 1950's with relatively large pools of unexploited ideas, many coming from abroad.

If we assume that stocks are evaluated at the beginning of the year and that employment is constant throughout the year, then the solution to (5) is:

$$y_t = A_{t-1} \begin{bmatrix} e^{\lambda_{1t-1}} \\ \vdots \\ e^{\lambda_{Mt-1}} \end{bmatrix},$$

for  $t = 1, \dots, 40$ . With  $y_0$  determined as described above, we can iterate forward using the equation  $A_{t-1} = x_{t-1} D_{t-1}$  where  $x_{t-1}$  is a matrix with columns equal to

the eigenvectors of  $\Delta_{t-1}$  (and associated eigenvalues,  $\lambda_{1t-1}, \dots, \lambda_{Mt-1}$ ) and  $D_{t-1}$  is a diagonal matrix with diagonal elements given by  $(x_{t-1})^{-1}y_{t-1}$ . The matrix  $\Delta_{t-1}$ , for  $t = 0, \dots, 40$ , is constructed from the parameters of the model, the work force in year  $t - 1$ , and research employment in year  $t - 1$ . The last 5 elements of  $y_t$  form the vector of stocks of diffused ideas in year  $t$ ,  $\mu_t$ . An index of labor productivity in country  $n$  in year  $t$  is simply  $A_{nt} = \mu_{nt}^{1/\theta}$ , for  $n = 1, \dots, 5$ .

#### 4.1 Baseline Simulation

The results of simulating the model are shown in table 3 and illustrated in figures 1 and 2. The table repeats the *data* on productivity (from table 1) in columns adjoining the values simulated by the *model* for each country. In 1950, data and model are equal by construction (given how we chose the 1950 value of  $\mu$ ). By 1990, data and model productivity levels remain surprisingly close, with the largest deviation being France where model productivity is 14% below actual productivity. Model productivity also captures the slowdown in productivity growth seen in the data for Germany, France, and Japan. As productivity levels in these countries approach levels in the United States, they benefit from a smaller fraction of U.S. ideas. Furthermore, over time the large pools of undiffused ideas fall, relative to levels of technology, and approach their steady-state levels. In contrast, the United States gets a slight boost to growth as its technology level declines relative to the others.

In Eaton and Kortum (1994) we chose parameters under the assumption that productivity in our five countries had reached a steady state by 1988. In fact, the results above suggest that this assumption was not too far off. In 1988, the simulated growth rates (calculated from 1988-1989) and relative levels of productivity

Table 3: Baseline Productivity Simulation

year	Germany		France		U.K.		Japan		U.S.	
	data	model	data	model	data	model	data	model	data	model
	(productivity levels)									
1950	.14	.14	.13	.13	.13	.13	.04	.04	.35	.35
1970	.45	.40	.41	.40	.29	.31	.25	.32	.57	.61
1980	.67	.58	.64	.55	.37	.42	.47	.48	.71	.80
1990	.86	.83	.91	.78	.66	.58	.78	.71	1.00	1.08
	(annual rates of growth, percent)									
1950-1970	5.9	5.4	5.7	5.5	3.9	4.3	9.0	10.1	2.4	2.8
1970-1980	4.2	3.6	4.3	3.3	2.4	3.1	6.2	4.1	2.2	2.7
1980-1990	2.4	3.6	3.6	3.4	5.8	3.1	5.1	4.0	3.5	3.0

The columns labeled *data* are value added per hour from table 1, while the columns labeled *model* are the values simulated by the model.

are,

	Germany	France	U.K.	Japan	U.S.
Growth rate (percent)	3.6	3.4	3.1	4.0	2.9
Relative productivity	.76	.71	.53	.65	1.00

The simulations indicate that by 1988 Germany, France, and the United Kingdom had all moved to within 80 per cent of their steady-state productivity position relative to the United States and were growing at about the steady-state rate. At the same time Japan was growing above its steady-state rate as its productivity level was still nearly 30 per cent below its steady state level relative to the United States. The United States was growing slightly below its steady-state rate in 1988, as it was not yet obtaining as many useful ideas from Japan as would eventually be coming its way.

## 4.2 Alternative Scenarios

We now consider some alternative counter-factual simulations. In table 4 we show how each alternative alters the simulated level of productivity in each country by 1990. In all cases the initial 1950 values of the state variables are the same as in

the baseline simulation. In examining these alternatives we continue to assume that the R&D effort in each country remained on its historical path.<sup>26</sup>

In the first alternative, 'complete isolation', we cut off diffusion between countries ( $\epsilon_{ni} = 0$  for  $n \neq i$ ) but we do not alter the diffusion rate within countries. Eliminating international diffusion has a devastating effect on productivity growth, with productivity in Germany, France, the United Kingdom, and Japan only about one-third of the baseline level by 1990. The United States fares relatively well since a sizable amount of research takes place within its borders. As a consequence, convergence of productivity to the U.S. level does not occur, although Japan catches up with the European countries.

The second alternative, 'complete integration', goes to the opposite extreme by removing the bias against innovations diffusing between countries relative to the speed at which they diffuse at home.<sup>27</sup> In this case, productivity is three to five times higher than the baseline level by 1990. Germany and France do particularly well, surpassing the level of productivity in the United States by 1990. Overall, productivity levels become more tightly clustered.

In the third alternative, 'U.S. isolated', we eliminate diffusion of technology between the United States and the other four countries. All countries are hurt: the United States because it obtains no ideas from abroad and the others because they obtain no ideas from the United States. The United States, of course, ends up with the same level of productivity as with 'complete isolation'. The other countries do relatively better since they still share ideas amongst each other. In

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<sup>26</sup>In fact, since our estimate of the elasticity of research output with respect to the input of research scientists and engineers ( $\beta$ ) is so tiny (.012) that productivity dynamics are quite insensitive to movements in this variable. While the effect of the changes we consider on R&D effort might be substantial, the implied effect on productivity would probably be very small.

<sup>27</sup>Thus, we set  $\epsilon_H = 9.77$  even for  $n \neq i$ .

Table 4: Alternative Scenarios: Productivity Levels in 1990

year	Germany	France	U.K.	Japan	U.S.
Data:					
1950	.14	.13	.13	.04	.35
1990	.86	.91	.66	.78	1.00
Simulations:					
Baseline	.83	.78	.58	.71	1.08
Alternatives:					
Complete isolation	.25	.23	.23	.24	.77
Complete integration	3.60	3.41	2.72	3.04	3.21
United States isolated	.64	.63	.49	.58	.77
Japan isolated	.63	.61	.46	.24	.96

The 'baseline' simulation is the same as in table 3. In 'complete isolation' we cut off diffusion between countries. In 'complete integration' we scale up diffusion between countries by the same factor we use to scale up diffusion within countries. In 'United States isolated' we cut off diffusion between the United States and other countries while in 'Japan isolated' we cut off diffusion between Japan and other countries.

the fourth alternative, 'Japan isolated', we cut off diffusion of technology between Japan and the other four countries. The European countries are hurt about as much as when they are isolated from the United States. Japan has the same level of productivity as with 'complete isolation' while U.S. productivity is 90 per cent of the baseline by 1990. A natural conclusion is that the free flow of ideas from either the United States or Japan to other industrialized countries is critical for world growth.

## 5 Conclusions

What can we conclude about the controversies posed in the introduction? The behavior of capital-output ratios indicate that technology differences, and not capital accumulation, explain differences in manufacturing productivity in the major industrial countries over the last four decades. In this paper we build a

model to examine the alternative view: that patterns of innovation and technology diffusion explain these differences.

As for the issue of foreign vs. domestic sources of growth, we conclude that growth is primarily the result of research performed abroad. We find that even the United States obtains over 40 per cent of its growth from foreign innovations. These findings seem to be consistent with historical accounts.<sup>28</sup>

Our model implies that, with international technological mobility, economies will converge to a steady state with parallel growth. Based on the initial conditions of 1950, we track quite closely the convergence of post-war manufacturing productivity in Germany, France, the United Kingdom, Japan, and the United States. Our interpretation is that this period was one of partial technological convergence from the great technological disparity left by World War II.

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<sup>28</sup>For an example of the importance of foreign technology, see Mueller's (1962) account of the foreign inventions underlying Du Pont's innovations.

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## A Mathematical Appendix

### A.1 The distribution of the technological frontier

Ideas are adopted in sector  $z$  at a stochastic rate of  $\dot{\mu}_{nt}z^{-\theta}$ . The probability that no idea is adopted in the time interval  $[t, t + dt]$  is thus  $e^{-\dot{\mu}_{nt}z^{-\theta}dt}$ . Therefore,

$$H_n(z|t + dt) = H_n(z|t)e^{-\dot{\mu}_{nt}z^{-\theta}dt}$$

or,

$$\frac{\partial \ln H_n(z|t)}{\partial t} = -\dot{\mu}_{nt}z^{-\theta}.$$

Solving this differential equation, with the two initial conditions: (i)  $\lim_{s \rightarrow -\infty} H_n(z|s) = 1 \quad \forall z \geq 1$  and (ii)  $\lim_{s \rightarrow -\infty} \mu_{ns} = 0$ , yields the cumulative distribution function for the technological frontier. The corresponding density is simply  $h_n(z|t) = \frac{dH_n(z|t)}{dz}$ .

### A.2 The geometric mean of the technological frontier

The log of the productivity index is simply,

$$\ln A_{nt} = \int_1^{\infty} \ln z h_n(z|t) dz.$$

Changing the variable of integration to  $x = \mu_{nt}z^{-\theta}$ ,

$$\ln A_{nt} = \theta^{-1} \int_0^{\mu_{nt}} \ln(\mu_{nt}/x) e^{-x} dx = \theta^{-1} \ln \mu_{nt} (1 - e^{-\mu_{nt}}) - \theta^{-1} \int_0^{\mu_{nt}} \ln x e^{-x} dx.$$

For large  $\mu_{nt}$  we have an arbitrarily good approximation,

$$\ln A_{nt} = \theta^{-1} \ln \mu_{nt} - \theta^{-1} \int_0^{\infty} \ln x e^{-x} dx.$$

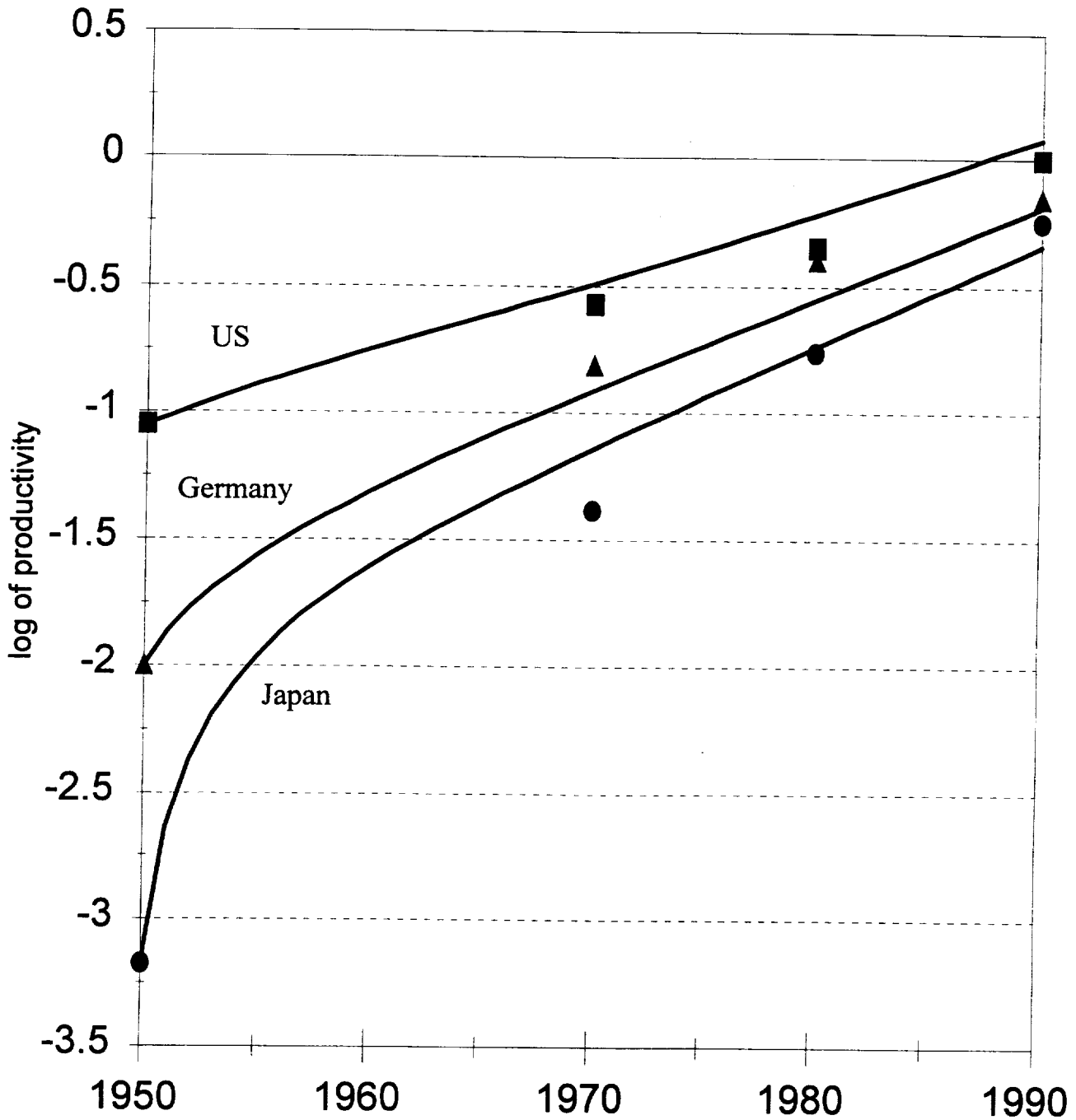
The Laplace transform of  $-\psi - \ln t$  is  $s^{-1} \ln s$ , where  $\psi$  is Euler's constant. Evaluating the Laplace transform at  $s = 1$  implies,

$$\int_0^{\infty} \ln x e^{-x} dx = -\psi.$$

This gives us the desired result that,

$$\ln A_{nt} = \theta^{-1} \ln \mu_{nt} + \psi/\theta.$$

**Fig. 1: Actual and Model Productivity**  
(actual in 1950, '70, '80, and '90)



**Fig. 2: Actual and Model Productivity**  
(actual in 1950, '70, '80, and '90)

